1. Which of the following subsets of \( \mathbb{R} \) have (length) volume zero.
   a) \( \mathbb{Q} \cap [0,1] \).
   b) \( \mathbb{Q}^c \cap [0,1] \)
   c) The Cantor set.

2. Let \( X \subset \mathbb{R}^n \) be a bounded subset which is contained in a proper vector subspace of \( \mathbb{R}^n \). Prove that \( X \) has volume zero.

3. Let \( X \subset \mathbb{R}^n \) be a set of volume zero and let \( T : \mathbb{R}^n \to \mathbb{R}^n \) be a linear map. Prove that \( T(X) \) has volume zero.

4. Let \( \Omega \subset \mathbb{R}^n \) be a connected region and let \( f : \Omega \to \mathbb{R} \) be a continuous function. Prove that there is a point \( c \in \Omega \) such that

\[
\int_{\Omega} f = f(c) \text{vol}(\Omega).
\]

5. Find the volume of the region in \( \mathbb{R}^3 \) bounded by the cylinders \( x^2 + y^2 = 1 \) and \( x^2 + z^2 = 1 \).

6. Use Fubini’s theorem to show that the mixed partial derivatives of \( C^2 \)-smooth function are equal.

7. \textbf{(Symmetry principle)} Let \( R \subset \mathbb{R}^n \) be a rectangle that is symmetric about the \( x_1 = 0 \) hyperplane, i.e., \((x_1, \ldots, x_n) \in R \implies (-x_1, x_2, \ldots, x_n) \in R \). Suppose \( f : R \to \mathbb{R} \) is an integrable function with the property that \( f(x_1, \ldots, x_n) = f(-x_1, x_2, \ldots, x_n) \). Compute \( \int_R f \). What if \( R \) were symmetric about the origin and \( f \) is an odd function? What about more general regions?

8. Let \( B \subset \mathbb{R}^3 \) be the open unit-ball. Evaluate

\[
\int_B x^2.
\]

9. Find the volume of the cone of radius \( r \) and height \( h \) using cylindrical coordinates and spherical coordinates.

10. Use the change of variables theorem to justify the use of polar, cylindrical and spherical coordinates.

   \textbf{Note:} Check the hypotheses of the change of variables theorem very carefully!