1. Let \( n \) be the last two digits of your roll number. Verify the series Fourier expansion of the entry numbered \( n \mod 20 \) in Table 1 of the textbook.

2. Recall that \( D_n \) was the \( n \)-th Dirchlet kernel. Define
   \[
   K_n := \frac{(D_0 + \cdots + D_n)}{n},
   \]
   the Fejer kernel. Compute \( K_n * f \).

3. Show that
   \[
   K_n(x) = \frac{1}{2\pi n} \frac{\sin^2(nx/2)}{\sin^2(x/2)}.
   \]

4. Solve Problems 5 and 6 on Page 43 of the textbook.

5. Solve Problem 12 on Page 48 of the textbook.

6. Solve problems 7 and 8 on Page 213 and use these to prove Theorem 2.7.

7. Compute the Fourier transform of \( f(x) = \frac{1}{x^2 + a^2}, a > 0 \) using the Calculus and residues and by using the inversion formula.

8. Show that the convergence of the Fourier series of \( f \) at a point \( x \) depends only on the behaviour of \( f \) near \( x \), i.e., if \( f(t) = g(t) \) for all \( t \) in some open interval containing \( x \) then the Fourier series of \( g \) converges to \( g(x) \) at \( x \) if and only if the Fourier series of \( f \) converges to \( f(x) \) at \( x \).

9. (The Schwartz Space) Let \( f : \mathbb{R} \to \mathbb{R} \) be continuous. We say \( f \) is rapidly decreasing at infinity if for each integer \( m > 0 \), the function \( |x|^m f(x) \to 0 \) as \( x \to \infty \). Let \( S \) denote the collection of all infinitely differentiable function all of whose derivatives are rapidly decreasing at infinity.
   a) Give several examples of functions in \( S \).
   b) Show that \( S \) is an algebra over \( \mathbb{R} \) with multiplication given by the usual product of functions.
   c) Show that the Fourier transform is well-defined on \( S \) and that \( \hat{f} \in S \).
   d) Show that \( S \) is an algebra under the operation \( \ast \).
   e) Let \( f \in S \) and let \( g = f + \hat{f} + \hat{\hat{f}} + \hat{\hat{\hat{f}}} \).

10. Show that
    \[
    \int_0^\infty \frac{\sin rx}{x} \, dx = \int_0^0 \frac{\sin rx}{x} \, dx = \frac{\pi}{2}.
    \]