Dynamic Mode Decomposition of Fluid Flow Data

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Outline

Introduction

- Accurate description of disturbance behaviour in complex flow simulations is of importance to
  1. Numerical simulations and physical experiments
  2. Understand the transition and instability mechanisms.
- POD determines the most energetic structures, but major short comings include
  1. The energy may not in all circumstances be the correct measure to rank the flow structures
  2. Phase information could not be extracted.

Dynamic Mode Decomposition

The transient data collected is represented in the form of a snapshot sequence

$$\mathbf{V}_1^N = (\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, ..., \mathbf{v_N}) \tag{1}$$

Assuming that linear mapping $\mathbf{A}$ connects the flow field $\mathbf{v_i}$ to the subsequent flow field $\mathbf{v_{i+1}}$ then

$$\mathbf{v_{i+1}} = \mathbf{A}\mathbf{v_i} \tag{2}$$

The sequence of flow fields can be formulated as a Krylov sequence as follows:

$$\mathbf{V}_1^N = (\mathbf{v_1}, \mathbf{A}\mathbf{v_1}, \mathbf{A}^2\mathbf{v_1}, ..., \mathbf{A}^{N-1}\mathbf{v_1}) \tag{3}$$

Our goal is the extraction of the dynamic characteristics such as

- eigenvalues, eigenvectors
- pseudoeigenvalues
- energy amplification
- resonance behaviour

of the dynamical process described by $\mathbf{A}$ based on the sequence $\mathbf{V}_1^N$

DMD contd...

In the limit we can express

$$\mathbf{v_N} = a_1\mathbf{v_1} + a_2\mathbf{v_2} + \ldots + a_{N-1}\mathbf{v_{N-1}} + \mathbf{r} \tag{4}$$

or compactly in matrix form we can express as

$$\mathbf{v_N} = \mathbf{V_1^{N-1}a} + \mathbf{r} \tag{5}$$

Consider

$$\mathbf{AV_1^{N-1}} = \mathbf{V_2^N} = \mathbf{V_1^{N-1}S} + \mathbf{re_{N-1}^T} \tag{6}$$

Where the matrix $\mathbf{S}$ is of companion type.
But the given snapshot sequence can be written as follows using singular value decomposition

$$\mathbf{V}_1^{N-1} = \mathbf{U\Sigma W}^H \tag{7}$$

DMD contd...

Robust implementation is achieved by constructing a full matrix $\tilde{\mathbf{S}}$

$$\tilde{\mathbf{S}} = \mathbf{U}^H \mathbf{V}_2^N \mathbf{W} \mathbf{\Sigma}^{-1} \tag{8}$$

Eigenvalues can be found using

$$\tilde{\mathbf{S}} \mathbf{y_i} = \mu_i \mathbf{y_i} \tag{9}$$

And Eigenvectors can be calculated using

$$\phi_i = \mathbf{U} \mathbf{y_i} \tag{10}$$

Coherence can be calculated by projecting specific dynamic modes $\phi_{\mathbf{i}}$ onto the POD basis $\mathbf{U}$.

Implementation Details

- Incompressible Navier-Stokes solver: *WenoHydro*
  - Fractional time step method
  - Staggered grid approach
  - 5^{th} order WENO scheme
  - RK3 for temporal discretiztaion
  - Smagorinsky, Vreman models with dynamic versions for LES.
- DMD code was implemented in Fortran 90.
- Eigenvalues , Eigenvectors are calculated using LAPACK libraries.
- Output files written in Tecplot format.

Applications - Lid Driven Cavity

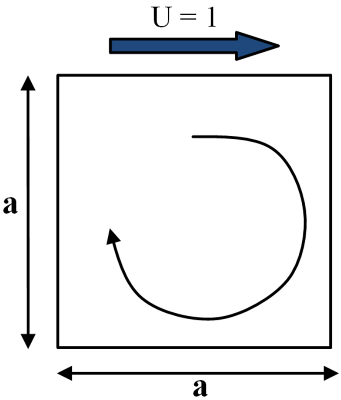


Figure: Schematic of lid driven cavity

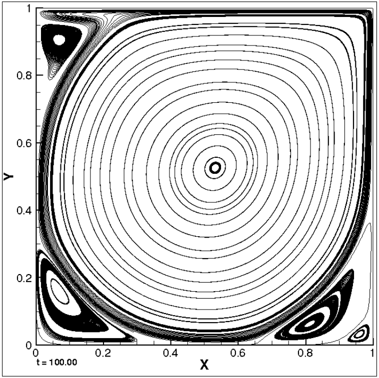
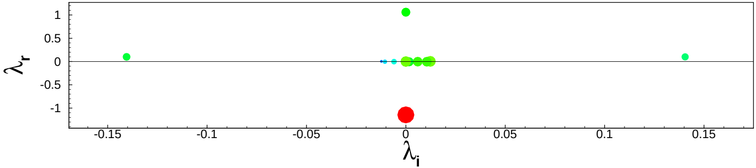
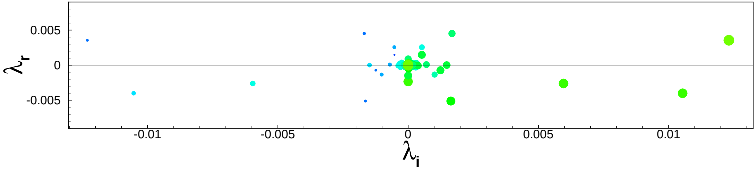


Figure: Stream lines and Vorticity animation

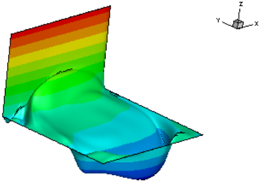
Lid Driven Cavity contd...

Reynolds Number	8500
Mesh Resolution	128x128
No. of snapshots	100
Smallest time scale	0.1

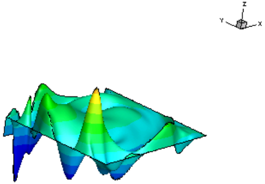




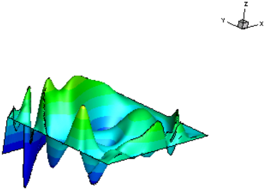
Lid Driven Cavity contd...



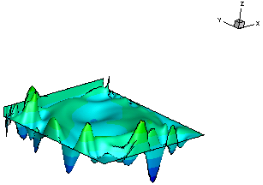
(a) $\lambda_r = 1.0600, \lambda_i = 0.0000$



(b) λ_r=0.00354, λ_i=0.0123

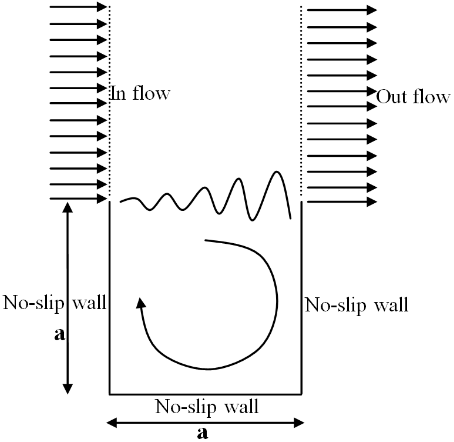


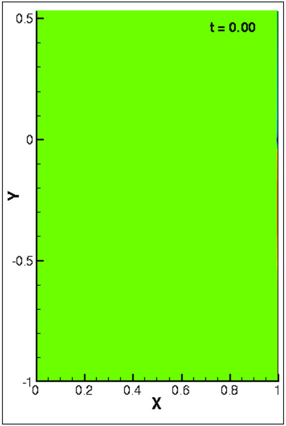
(c) λ_r=0.00088, λ_i=0.0000



(d) λ_r=0.00027, λ_i=0.00009

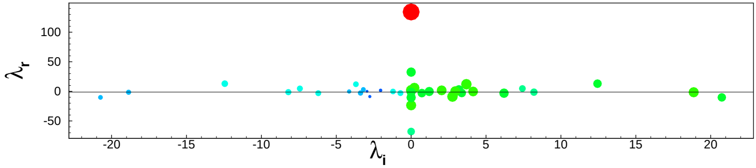
Applications - Open Cavity

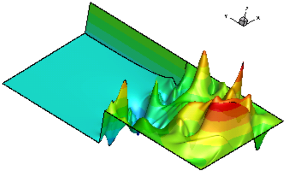




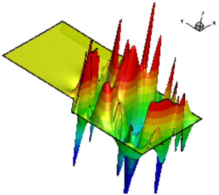
Open Cavity contd...

Reynolds Number	4500
Mesh Resolution	512x1024
No. of snapshots	50
Smallest time scale	0.025

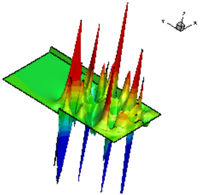




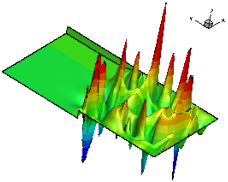
(a) $\lambda_r = 134.2, \lambda_i = 0.0$



(b) $\lambda_r = 12.05, \lambda_i = 3.68$



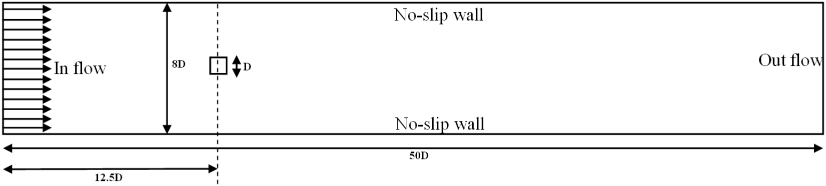
(c) $\lambda_r = 3.09, \lambda_i = 3.19$

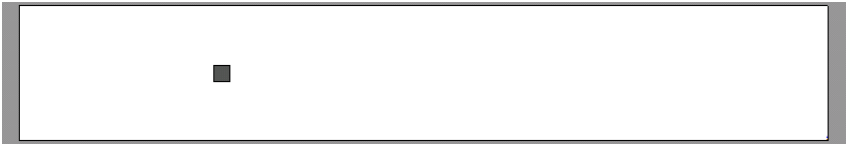


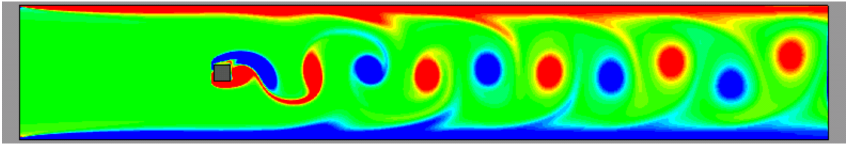
(d) $\lambda_r = 0.14, \lambda_i = 2.95$

Figure: Dynamic Modes visualized by the streamwise velocity component

Applications - Flow Past a Square Cylinder in a Channel

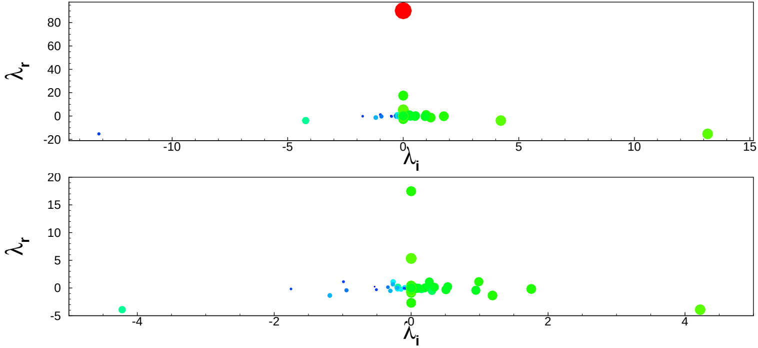




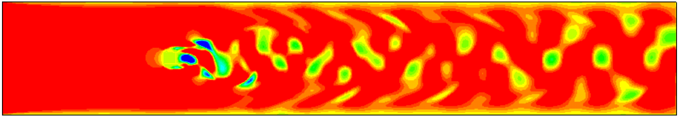


Flow Past a Square Cylinder contd...

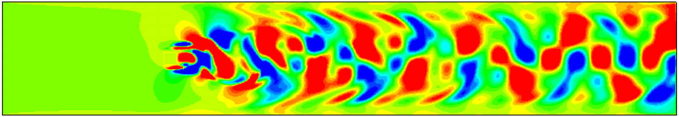
Reynolds Number	200
Mesh Resolution	1024x128
No. of snapshots	100
Smallest time scale	0.1



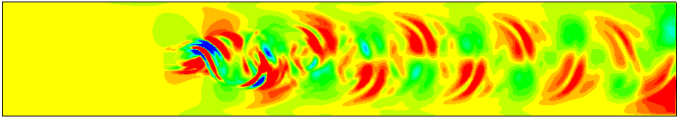
Flow Past a Square Cylinder contd...



(a) $\lambda_r = 90.0, \lambda_i = 0.00$



(b) $\lambda_r = 17.45, \lambda_i = 0.00$



(c) $\lambda_r = 5.33, \lambda_i = 0.00$

Figure: Dynamic Modes visualized by non-zero component of vorticity

Summary

- The DMD was implemented and test on three different flow configurations.
- It appears to be capable of extracting dominant flow features from snapshots.
- While POD focuses on representation based on spatial orthogonality, DMD focuses on a representation based on temporal orthogonality (frequencies)
- Can be applied equally well to simulations, and experimental data.

References

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- [Henningson, D. S.] *Description of complex flow behaviour using global dynamic modes*, J. Fluid Mech. (2010).
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- [Holmes, P., Lumley, .J. L., Berkooz, G.] *Turbulence, coherent structures, dynamical systems and symmetry.* Cambridge University Press 1996.
- [Sirovich, L.] Turbulence and the dynamics of coherent structures. Part I-III. Quart J. Appl. Math. 1989.

Thank You!
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Happy Holidays...