Dynamic Mode Decomposition of Fluid Flow Data

Kameswararao Anupindi

School of Mechanical Engineering
Purdue University

December 14, 2010
Outline

1 Introduction
2 Dynamic Mode Decomposition (DMD)
3 Implementation Details
4 Applications and Results
   ▶ Lid Driven Cavity
   ▶ Open Square Cavity
   ▶ Square Cylinder in a Channel
5 Summary
6 References
Accurate description of disturbance behaviour in complex flow simulations is of importance to

1. Numerical simulations and physical experiments
2. Understand the transition and instability mechanisms.

POD determines the most energetic structures, but major short comings include

1. The energy may not in all circumstances be the correct measure to rank the flow structures
2. Phase information could not be extracted.
Dynamic Mode Decomposition

The transient data collected is represented in the form of a snapshot sequence

$$V_1^N = (v_1, v_2, v_3, ..., v_N)$$ (1)

Assuming that linear mapping $A$ connects the flow field $v_i$ to the subsequent flow field $v_{i+1}$ then

$$v_{i+1} = Av_i$$ (2)

The sequence of flow fields can be formulated as a Krylov sequence as follows:

$$V_1^N = (v_1, Av_1, A^2v_1, ..., A^{N-1}v_1)$$ (3)

Our goal is the extraction of the dynamic characteristics such as

- eigenvalues, eigenvectors
- pseudoeigenvalues
- energy amplification
- resonance behaviour

of the dynamical process described by $A$ based on the sequence $V_1^N$. 
In the limit we can express

\[ \mathbf{v}_N = a_1 \mathbf{v}_1 + a_2 \mathbf{v}_2 + \ldots + a_{N-1} \mathbf{v}_{N-1} + \mathbf{r} \]  \hspace{1cm} (4)

or compactly in matrix form we can express as

\[ \mathbf{v}_N = \mathbf{V}_1^{N-1} \mathbf{a} + \mathbf{r} \]  \hspace{1cm} (5)

Consider

\[ \mathbf{A} \mathbf{V}_1^{N-1} = \mathbf{V}_2^N = \mathbf{V}_1^{N-1} \mathbf{S} + \mathbf{r} \mathbf{e}_{N-1}^T \]  \hspace{1cm} (6)

Where the matrix \( \mathbf{S} \) is of companion type.
But the given snapshot sequence can be written as follows using singular value decomposition

\[ \mathbf{V}_1^{N-1} = \mathbf{U} \mathbf{\Sigma} \mathbf{W}^H \]  \hspace{1cm} (7)
Robust implementation is achieved by constructing a full matrix $\tilde{S}$

$$\tilde{S} = U^H V_2^N W \Sigma^{-1}$$  \hspace{1cm} (8)

Eigenvalues can be found using

$$\tilde{S}y_i = \mu_i y_i$$  \hspace{1cm} (9)

And Eigenvectors can be calculated using

$$\phi_i = U y_i$$  \hspace{1cm} (10)

Coherence can be calculated by projecting specific dynamic modes $\phi_i$ onto the POD basis $U$. 
Implementation Details

- Incompressible Navier-Stokes solver: *WenoHydro*
  - Fractional time step method
  - Staggered grid approach
  - 5\textsuperscript{th} order WENO scheme
  - RK3 for temporal discretization
  - Smagorinsky, Vreman models with dynamic versions for LES.

- DMD code was implemented in Fortran 90.
- Eigenvalues, Eigenvectors are calculated using LAPACK libraries.
- Output files written in Tecplot format.
Applications - Lid Driven Cavity

Figure: Schematic of lid driven cavity

Figure: Stream lines and Vorticity animation
Lid Driven Cavity contd...

Reynolds Number 8500
Mesh Resolution 128x128
No. of snapshots 100
Smallest time scale 0.1

Figure: DMD spectrum for lid driven cavity flow at Re = 8500

Kameswararao Anupindi (Purdue University)
ME611, Principles of Turbulence
Lid Driven Cavity contd...

(a) $\lambda_r = 1.0600, \lambda_i = 0.0000$  
(b) $\lambda_r = 0.00354, \lambda_i = 0.0123$

(c) $\lambda_r = 0.00088, \lambda_i = 0.0000$  
(d) $\lambda_r = 0.00027, \lambda_i = 0.00009$
Applications - Open Cavity

In flow

Out flow

No-slip wall

No-slip wall

No-slip wall

No-slip wall
Reynolds Number 4500
Mesh Resolution 512x1024
No. of snapshots 50
Smallest time scale 0.025
Open Cavity contd...

(a) \( \lambda_r = 134.2, \lambda_i = 0.0 \)  
(b) \( \lambda_r = 12.05, \lambda_i = 3.68 \)

(c) \( \lambda_r = 3.09, \lambda_i = 3.19 \)  
(d) \( \lambda_r = 0.14, \lambda_i = 2.95 \)

Figure: Dynamic Modes visualized by the streamwise velocity component
Applications - Flow Past a Square Cylinder in a Channel

In flow

No-slip wall

Out flow

No-slip wall

12.5D

8D
Flow Past a Square Cylinder contd...

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds Number</td>
<td>200</td>
</tr>
<tr>
<td>Mesh Resolution</td>
<td>1024×128</td>
</tr>
<tr>
<td>No. of snapshots</td>
<td>100</td>
</tr>
<tr>
<td>Smallest time scale</td>
<td>0.1</td>
</tr>
</tbody>
</table>

![Graph showing data points and axes labeled $\lambda_r$ and $\lambda_i$.](image-url)
Flow Past a Square Cylinder contd...

(a) $\lambda_r = 90.0, \lambda_i = 0.00$

(b) $\lambda_r = 17.45, \lambda_i = 0.00$

(c) $\lambda_r = 5.33, \lambda_i = 0.00$

**Figure:** Dynamic Modes visualized by non-zero component of vorticity
The DMD was implemented and tested on three different flow configurations.

It appears to be capable of extracting dominant flow features from snapshots.

While POD focuses on representation based on spatial orthogonality, DMD focuses on a representation based on temporal orthogonality (frequencies).

Can be applied equally well to simulations, and experimental data.


Thank You!
&
Happy Holidays...