# Transmission Photoelasticity

## Birefringence or Double Refraction

Birefringence is defined as the double refraction of light in a transparent, molecularly ordered material, which is manifested by the existence of orientation-dependent differences in refractive index.

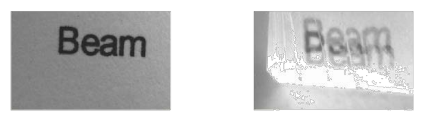



Fig.1 Double Refraction in a Calcite crystal

Figure 1 shows the phenomenon of birefringence in a Calcite crystal. Anisotropic crystals, such as quartz, calcite, and tourmaline exhibit permanent birefringence. When light enters the optical axis of anisotropic crystals, it behaves in a manner similar to the interaction with isotropic crystals, and passes through at a single velocity. However, when light enters through a non-equivalent axis, it is refracted into two rays, each polarized with the vibration directions oriented mutually perpendicular to one another and travelling at different velocities causing double refraction (Fig.1).

## Physical Principle used in Photoelasticity – a brief discussion

Certain non crystalline transparent materials notably some polymers, are optically isotropic under normal conditions but become doubly refractive/birefringence when stressed. This effect normally persists while the loads are maintained but vanishes almost instantaneously or after some interval of time after removal of load. This phenomenon of temporary or artificial birefringence was first noted by Brewster in 1816 and the physical characteristic of photoelasticity is based on this.

This property of these materials is exploited to determine stress field within a model using the "Stress - optic law". Formulated by Maxwell in 1852 the Stress optic law states that,

$$(\sigma_1-\sigma_2)=\frac{NF_\sigma}{h} \qquad (1)$$

Where,

$(\sigma_1-\sigma_2)$ — Principle stress difference (MPa)
$N$ — Fringe order
$h$ — Thickness of the specimen (mm)
$F_\sigma$ — Material stress fringe value (N/mm/fringe)

Equation (1) implicitly gives the indication that $F_\sigma$ and $(\sigma_1-\sigma_2)$ are linearly related. However, at higher stress levels, the relationship is non-linear and Eq. (1) cannot be used. Therefore, Eq. (1) should be used with care.

## Model to Prototype Relations

One of the first questions that come up in anybody's mind is how does one relate the results from conducting experiment on plastics to metallic prototypes?

The validity can be explained from considering the nature of the equations that are necessary for solving the stress distribution. In 2D problems, the equations of equilibrium together with the boundary conditions and the compatibility conditions in terms of stress components give us a system of equations that are usually sufficient for complete determination of the stress distribution. The equilibrium equations are,

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+F_x=0 \qquad (2)$$

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+F_y=0 \qquad (3)$$

The compatibility conditions for plane stress and plane strain conditions in terms of stress components are,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\sigma_x+\sigma_y)=-(1+\nu)\left(\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}\right) \qquad (4)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\sigma_x+\sigma_y)=-\left(\frac{1}{(1-\nu)}\right)\left(\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}\right) \qquad (5)$$

It is interesting to note that in the case of constant body force, the equations determining the stress distribution do not contain material elastic constants. Thus, the stress magnitude and their distribution are same for all isotropic materials. However, the strains and also deformations are functions of elastic constants. Equations (2) to (5) also indicate that the stress distribution in the elastic state is independent of the loads and the scale of the model.

Thus, for making photoelastic models, the scale of the models and loads may be chosen as convenient. The model results can be related to the prototypes by following the laws of similarity, provided, the model and the prototypes are geometrically similar and the distribution of the loading is the same for both. The similarity relations do not apply near points of loading and the principle of Saint Venant is usually invoked for other regions.

## Polarization of light

In photoelasticity, light is used as a sensor. Typically for a known incident light, the emergent light is analysed for stress information. Light waves are transverse waves having two vector fields namely electrical and magnetic. Most light sources consist of a large number of randomly oriented atomic or molecular emitters. The light rays emitted from such a source will have no orientation and the tip of the light vector describes a random vibratory motion in a plane transverse to the direction of propagation. If the tip of the light vector is forced to follow a definite law, the light is said to be polarised.

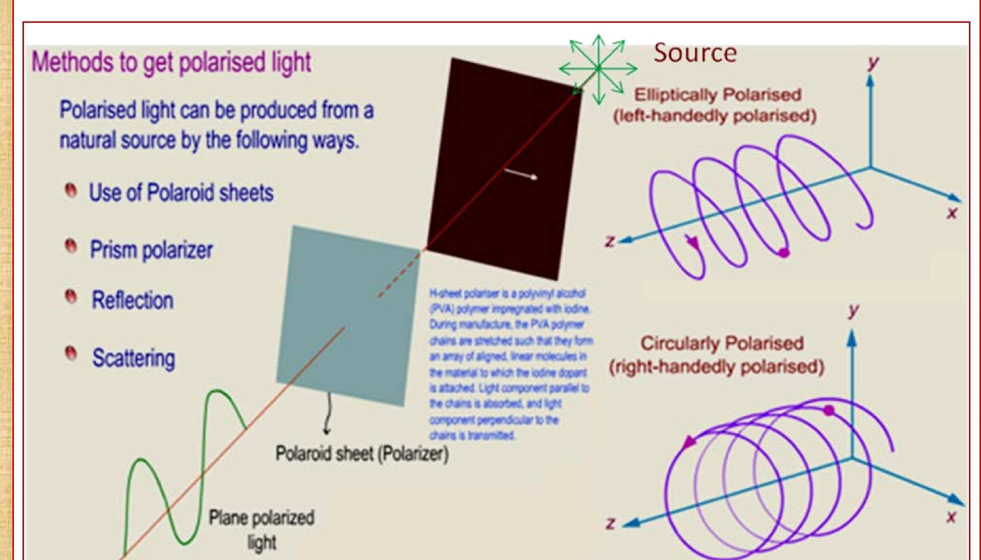



Fig. 2 Polarization of light

One uses polarized light to visualize a photoelastic model for ease of data interpretation. The instrument used to observe photoelastic phenomenon is called a Polariscope.

## Components of Polariscope

In a transmission polariscope, light is transmitted through the model. The polariscope can either be a lens or diffused light type. Figure 3 shows the components used in transmission photoelasticity.

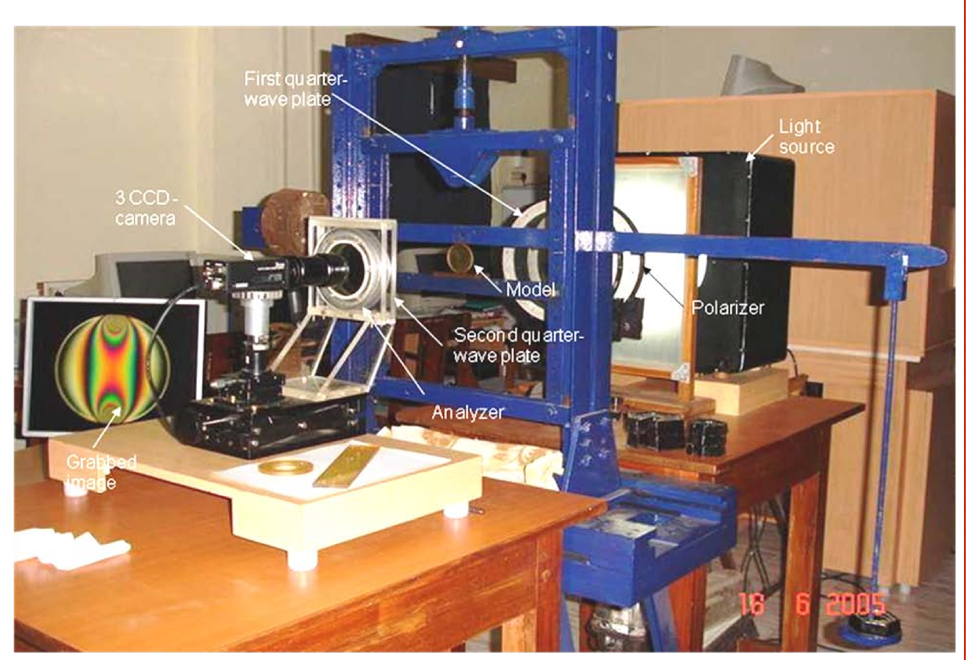



Fig. 3 Components of a Modern Transmission Polariscope

## Plane Polariscope

One of the simplest optical arrangements possible is a plane polariscope. Figure 4 shows the optical arrangement of a plane polariscope. The incident light on the model is plane polarised. As it passes through the model, the state of polarization changes from point to point depending on the principal stress direction and the magnitude of principal stress difference. The information about the stress field can be obtained if the state of polarization of the emergent light is studied. This is easily achieved by introducing a polarizer at 0 . Since, this optical element helps to analyse the emergent light, it is known as analyzer.

With the introduction of an analyzer, fringe contours appear black on the screen. The fringe contours correspond to those points where intensity of light transmitted is zero. Since the analyzer is kept at 0 , this is possible only if the emergent light from the model has its plane of polarization along the vertical.

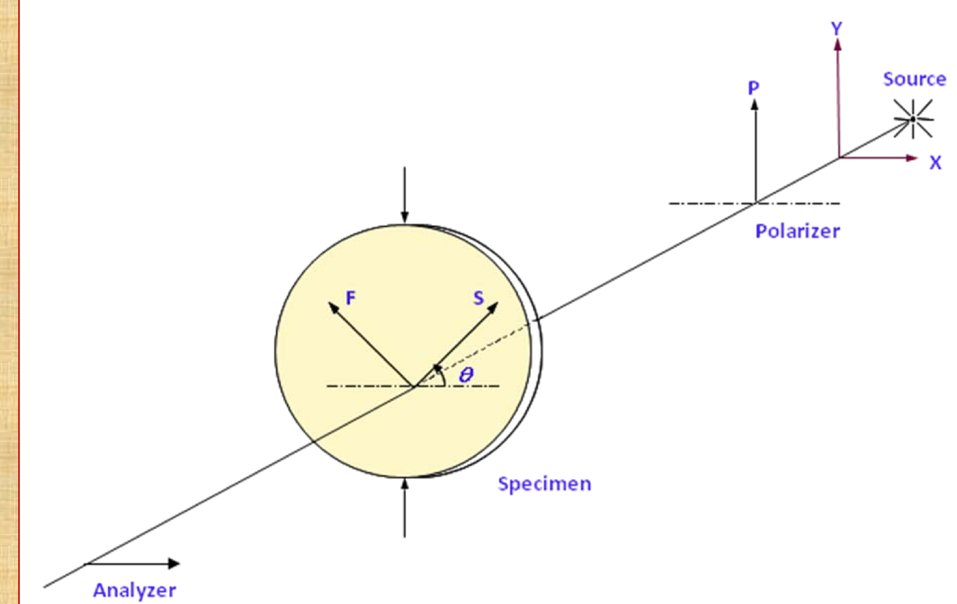



Fig. 4 Optical arrangement of a plane polariscope

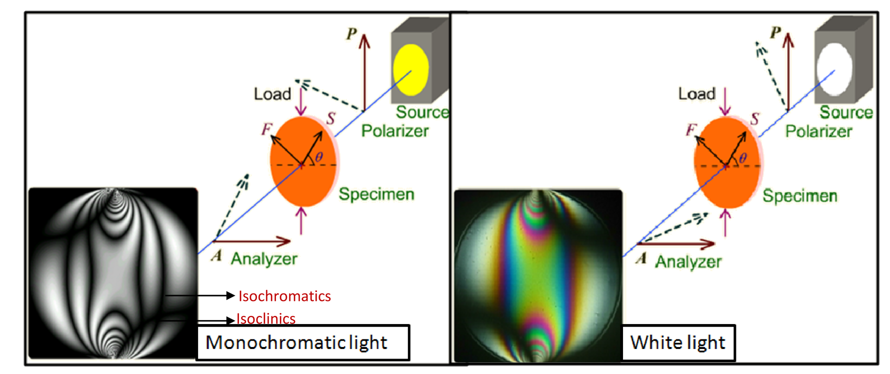



Fig. 5 Fringe patterns observed in plane polariscope under monochromatic light and white light (dark field)

Thus, to give physical meaning to the fringe contours, we have to identify the conditions under which the incident plane polarized light is unaltered as it passes through the model.

The incident light is unaltered on all those points where the model behaves like a full wave plate. This happens when the principal stress difference is such as to cause a relative phase difference of $2m\pi$ ($m$ = 0, 1, 2 ...), where $m$ is an integer. Since stress is continuous, one observes a collection of points forming contours that satisfy the above condition and the respective fringe field is known as *Isochromatics*.

Iso means constant and chroma means colour. Coloured contours are seen only when the illuminated source is white light. By knowing isochromatic fringe order at a point and using stress optic law, one can determine the principal stress difference at the point.

Another possibility, wherein the incident light is unaltered is, when the polarizer axis coincides with one of the principal stress directions at the point of interest. In this case, light extinction is not wavelength dependent and one observes a dark fringe even in white light. These are known as isoclinics meaning contours of constant inclination. The principal stress direction on all points lying on an isoclinic is a constant and is indicated in Fig. 6.

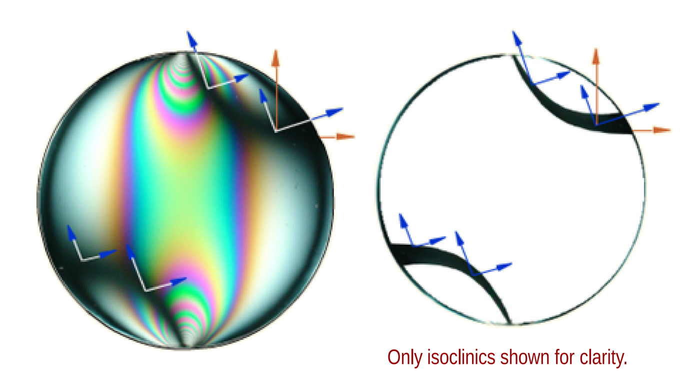



Fig. 6 Fringe patterns observed in plane polariscope under white light (dark field)

Thus, in plane polariscope, one has two sets of contours, namely, isochromatics and isoclinics superposed over each other. Leaving the zeroth fringe order isochromatics, these two contours can be distinguished by using white light. To distinguish an isoclinic from a zeroth fringe order, rotate the polarizer and analyzer in crossed combination. The zeroth fringe order will not move whereas the isoclinics will move.

## Circular Polariscope

Figure 7 shows the optical arrangement of a circular Polariscope. In this a circularly polarised light is used to reveal the stress information in the model. This is achieved by introducing a quarter-wave plate with one of its axis at 45 after the polarizer.

The emergent light is analysed using a combination of a quarter-wave plate and the analyzer. Inside the model boundary, the intensity of light transmitted is governed by the stress field and this can be obtained by Jones calculus.

In dark- field arrangement, intensity is zero when $\delta = 2m\pi$ ($m$ = 0, 1, 2...) and fringes correspond to 0, 1, 2 ... etc. In bright-field arrangement intensity is zero when $\delta = 2(m+1)\ \pi$, i.e., when the retardation is an odd multiple of half- wave lengths and the fringes correspond to 0.5, 1.5, 2.5 etc.

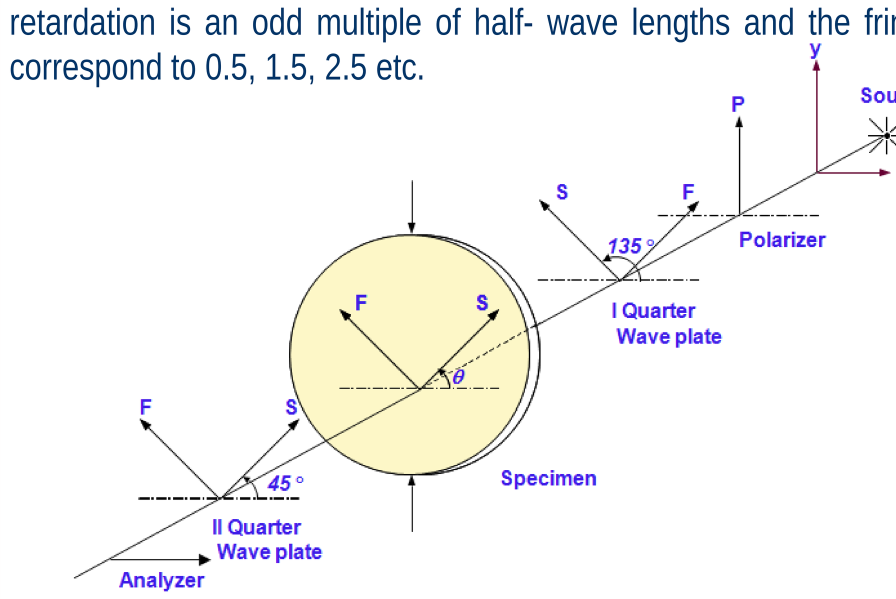
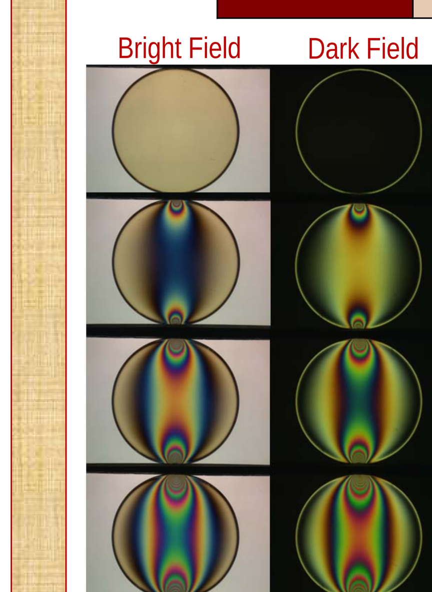



Fig. 7 Optical arrangement of a Circular polariscope

Table 1 shows the various conventional optical arrangements to set dark and bright field.

Table 1. Various Optical arrangements

| Set up | Polarizer and analyzer | Quarter wave plates | Back ground |
|---|---|---|---|
| Plane polariscope | Crossed | None | Dark |
| Circular Polariscope | Crossed | Crossed* | Dark |
| | Parallel | Parallel | Dark |
| | Crossed | Parallel | Bright |
| | Parallel | Crossed* | Bright |




Quarter wave plates crossed arrangements are preferred as it minimizes the influence of the quarter wave plate error. Figure 8 shows formation of Isochromatic fringes as load increases gradually from zero to maximum (up to $N$ = 3).

Fig. 8 Fringe patterns in a circular polariscope under white light

## Ordering of Isochromatics

Ordering of isochromatics is one of the crucial steps in the process of determining quantitative information from the fringe field. Experience and intuition have guided the numbering of isochromatic fringes and it is not an easy task to order the fringes from a given dark/bright field black and white image. One of the simplest approaches is to use a white light source, as the colour code can be used as given in Table 2 to identify the fringe orders and also the fringe gradient direction (fig.9).

Table 2. Sequence of colours produced in a dark- field

| Colour Observed | Retardation (nm) | Approximate Fringe Order |
|---|---|---|
| Black | 0 | 0.00 |
| Grey | 160 | 0.28 |
| White | 260 | 0.45 |
| Yellow | 350 | 0.60 |
| Orange | 460 | 0.79 |
| Dull Red | 520 | 0.90 |
| Tint of Passage | 577 | 1.00 |
| Blue | 620 | 1.06 |
| Blue-Green | 700 | 1.20 |
| Green- Yellow | 800 | 1.39 |
| Orange | 940 | 1.63 |
| Rose Red | 1050 | 1.82 |
| Tint of Passage | 1150 | 2.00 |
| Green | 1350 | 2.35 |
| Green-Yellow | 1450 | 2.50 |
| Orange- red | 1550 | 2.65 |
| Tint of Passage | 1730 | 3.00 |

The method is advantageous up to the third order fringe in the field, as beyond this the colours merge. Another question is how to determine fringe order at a point to interest through which no bright or dark field fringe passes through but the fringe gradient information is known.

Method (1)

One of the simplest methods is by using a dark-field set up and a bright-field set up. Two sets of isochromatic fringe patterns over the field of view are recorded. The dark field set up gives fringes representing an integral number of fringe orders. The bright field set up gives fringes representing an odd multiple of half orders.

With these, one can determine the retardation existing at the point graphically.

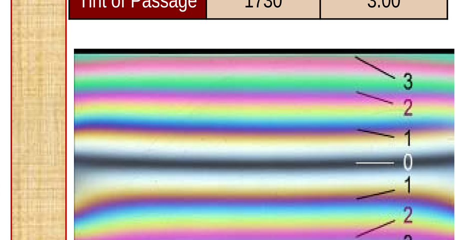



Fig. 9 Isochromatic fringes observed for a four point bend specimen

Method (2) - Compensation Techniques

Compensation techniques are basically point by point techniques. The basic principle is that, by external means, the retardation provided by the model is compensated such that a fringe passes through the point of interest. The additional retardation added or subtracted is known as fractional retardation. Analyzer can be used a compensator. The use of analyser as a compensator is known as Tardy's method of compensation.

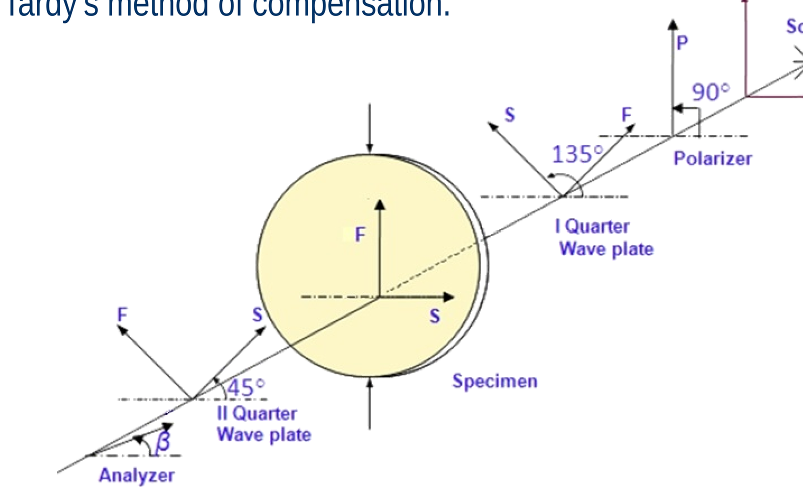



Fig. 10 Optical arrangement for Tardy's method of compensation

Procedure: The principal stress directions at the point of interest are determined using a plane polariscope.

A circular polariscope is then formed such that the polarizer is kept at the isoclinic angle and all the other optical arrangements are appropriately arranged. The analyzer alone is then rotated until a fringe passes through the point of interest. The rotation given to the analyzer can be related to the fractional retardation as,

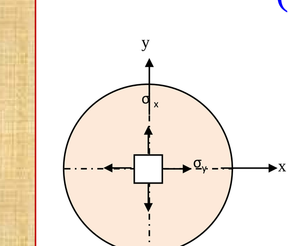
$$\delta_n=\pm\frac{|\beta|}{180} \qquad (6)$$

## Calibration of Photoelastic Model Material

The stress-fringe value varies with time and also from batch to batch. Hence, it is necessary to calibrate each sheet of casting or sheet at the time of the test. Calibration is performed on simple specimens whose closed formed stress field solution is known. Although the stress fields for simple tension or beam under pure bending is known, the use of circular disc under diametral compression is preferred for calibration. This is because the specimen is compact, easy to machine and can also be loaded easily.

The principal stress difference at any point in the disc can be expressed as,

$$(\sigma_1-\sigma_2)=\left(\frac{4PR}{\pi t}\right)\left(\frac{R^2-(x^2-y^2)}{\left(x^2+y^2+R^2\right)^2-(4y^2R^2)}\right) \qquad (7)$$




At the centre of the disc due to symmetry, the shear stress is zero and the principal stress difference is obtained as,

$$(\sigma_1-\sigma_2)=\left(\frac{8P}{\pi Dt}\right) \qquad (8)$$

From Eq. (1) and Eq. (8) the material stress fringe value can be obtained as

$$(F_\sigma)=\left(\frac{8P}{\pi DN}\right) \qquad (9)$$

$N$ at the centre of the disc can be determined using Tardy's method of compensation.

## Specimen handling and Maintenance

Photoelastic specimens should be stored in vacuum desiccator with samples wrapped using tissue paper. The desiccator base is filled with silica gel to absorb moisture and lid - base interface is greased to prevent fresh air/ moisture from entering. When handling samples one should not touch the corners as it induces spurious fringes know as time edge effect (Fig.11).

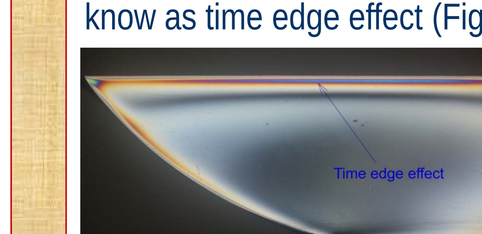



Fig. 11 Specimen with time edge effect

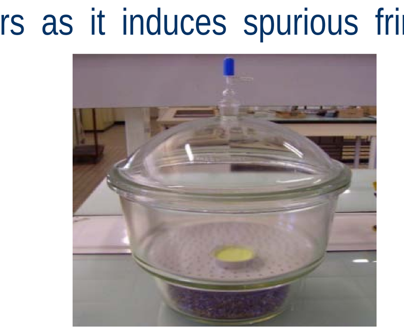


Fig. 12 Desiccator with silica gel in the base

## Reference


[1] K Ramesh: e-book on Experimental Stress Analysis, IIT Madras (2009). ISBN: 978- 81-904235-6-4, http://apm.iitm.ac.in/smlab/kramesh/book_5.htm

[2] "Photoelasticity" author – Krishnamurthi Ramesh, in Sharpe (Ed.) [Chapter 25, Springer Handbook of Experimental Solid Mechanics], Springer, New York, USA, 701-742, (2008).



Courtesy: e-book on Experimental Stress Analysis, Prof. K Ramesh; NPTEL Experimental Stress Analysis (Video lecture series), Prof. K Ramesh

Gift from *Prof. K. Ramesh*, Digital Photomechanics Laboratory, IIT Madras, India