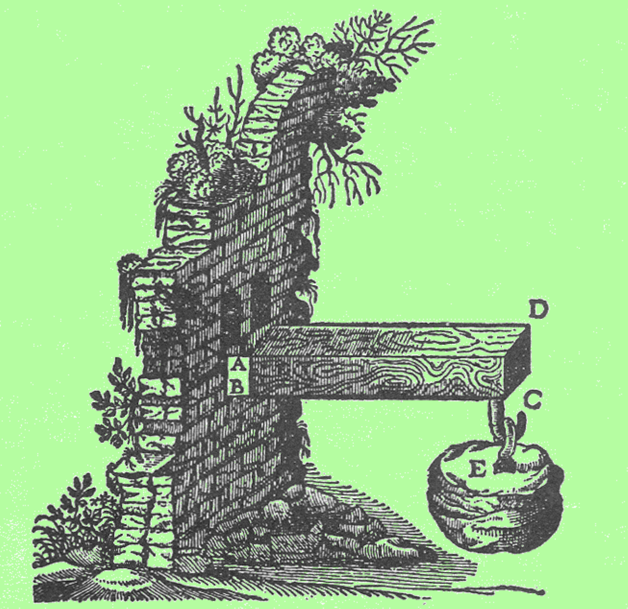
Strength of Materials
Laboratory Manual



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Preface

Welcome to the exciting world of experimenting on the strength of materials. Great strides in the world of mechanics have been achieved only through keen experimental observations. Translating experimental observations to meaningful aspect of analytical understanding is not a simple task. Even great minds in the field of mechanics have committed blunders and it took 200 years to develop an acceptable theory on the deflection of beams!

In a first course on Mechanics of Solids one learns the stress distribution on slender members subjected to simple loading configurations. The cross-section of the slender member and the loadings are carefully chosen such that plane sections remain plane before and after loading. This considerably simplifies the analytical approach and thus you read torsion of circular cross-sections and bending of rectangular beams / I beams etc. In this booklet, experiments on slender members only are presented. The choice of dimensions of the slender member is so chosen such that the assumptions made in analytical modeling are closely satisfied.

It is desirable to perform experiments after one learns the theory behind it. Since, this lab course runs concurrently with the course on Strength of Materials, conscious effort is made to present each experiment intelligible to a student who has no such advantage. This has necessitated including more information in each experiment, sometimes amounting to pre-conditioning the inquisitive mind. Each experiment ends with points for discussions and the 'starred' questions are to be submitted at the end of the semester as one needs theoretical background as well to answer them.

It is worthwhile to note that in analytical development, torsion of rectangular cross-section is postponed to a second level course in Mechanics of Solids primarily because plane sections do not remain plane before and after loading! In beams, the flexure formula is valid only for a beam subjected to pure bending. For all other cases, it is only approximate and the books term the analysis for such beams as Engineering analysis of beams. This is because, the presence of shear force violates the assumption that plane sections remain plane before and after loading. Nevertheless, the solution based on flexure formula is acceptable from an engineering standpoint. On the contrary experimental measurement of these complex problems are straight forward and represents truth.

Several postgraduate students of this laboratory have enthusiastically participated in the preparation of this booklet and also setting up the experiments listed here. My special thanks are to these students and also to the laboratory staff namely Mr. A. Sadasivam, Mr. A. Antony Raj and Mr. T. S. Elumalai in helping me to set up the experiments.

Prof. K. Ramesh
July 2003

General Guidelines

- Please read the details of the experiment given in the booklet thoroughly before you come for the laboratory class.
- Come prepared with tables to enter raw data pertaining to your experiment. Necessary guidelines to prepare the table for raw data is mentioned in this booklet.
- The table of raw data should be extensive and should contain all details of the experiment including the initial readings so that any error in measurement could be checked even at a later date.
- Raw data is to be entered in ink and at the end of the experiment get the signature of the lab-incharge on it.
- There will be one report per group.
- One person has to write it and the other two must check it. Doing correct calculations is important in this lab course. Otherwise one will not be able to draw right conclusions. Difference of 10% in results is easily acceptable in experimental work. Any mistake in calculation means – heavy penalty to checkers only.
- If you miss a lab, you will have to do the experiment separately and write the report separately too.

Lab Report :- Lab report has to be formal, neatly written on good sheets in legible handwriting.

(i) **Flower Garland Analogy** :- A technical write-up can be compared with a flower garland whose flowers are figures, graphs, pictures and equations. Your text works as a thread. So, prepare your tables, figures and graphs before you start writing.

(ii) **Graphs** :- They are the most important elements of the report. A reader can easily grasp the results through well prepared graphs. Graphs, figures and photographs must be numbered as Fig.1, Fig.2, etc. with appropriate captions. The axes should be defined completely. If there are more than one line on the graph, each line should be identified clearly. Each figure should be called by the text.

Tables : Tables are also to be numbered and for each table give a title. Moreover a table should be readable. Do not bring unnecessary details from the raw data.

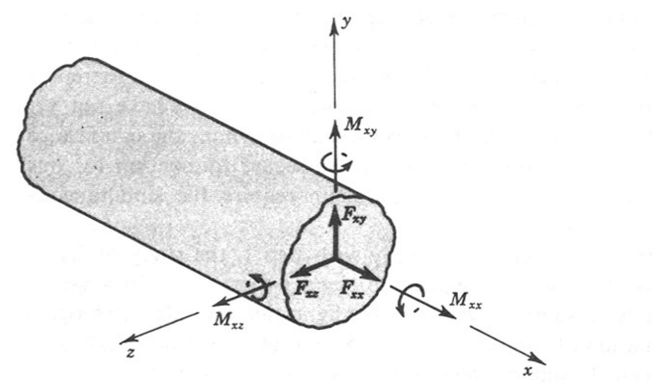
Precise and to the point : A report should be precise but complete. Theory and experimental details should be to the point. I do not want to see essays in theory and experimental details

Units : Only S.I. units please. There is a separate unit for a quantity. Force should be represented in N, stress in MPa and modulus generally in GPa.

Tampering with data : Biggest crime in technical work is if you tamper with data. I will accept any absurd data and still give you good marks. May be our instrument has gone bad on that particular day and you are getting faulty readings. However, if I find you cheating – all the members of the group will get zero in that report.

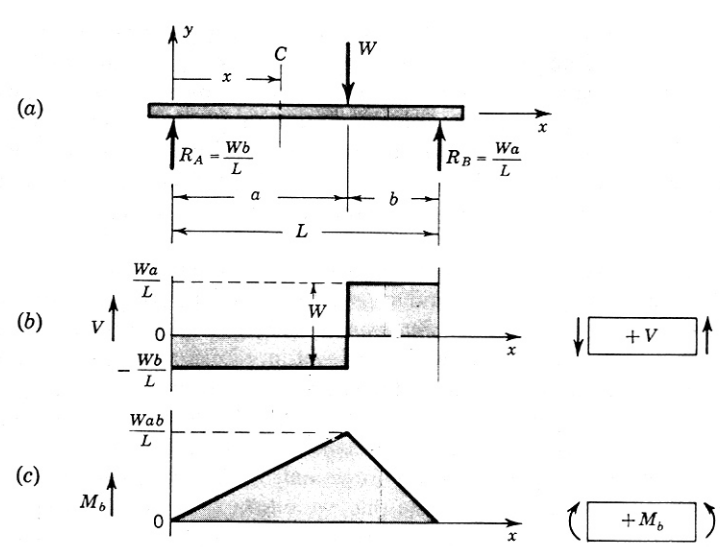
Sign Convention

Imagine a *hypothetical cut* or *section* across the slender member. If either part of the member is considered as an isolated free body, the force and moment required at the section to keep that part of the member in equilibrium can be obtained by applying the conditions of equilibrium. In general, there will be both a force and moment acting across the section as shown in the figure below.



The notation F_{xx} , …, etc., of the components in the figure is used to indicate both the orientation of the cross section and the direction of the particular force or moment component. The first subscript indicates the direction of the outwardly directed normal vector to the face of the cross section. The cross-sectional *face* is considered *positive* when the outward normal points in a positive co-ordinate direction and *negative* when its outwardly directed normal vector points in the negative coordinate direction. The second subscript indicates the coordinate direction of the force or moment component. The equations presented in this booklet follow the above sign convention.

In drawing the shear force diagram and bending moment diagrams various sign conventions are used by different authors. The following sign convention is adopted in this booklet and the flexure formula needs to be interpreted based on this sign convention.



Force or moment in the positive direction on the positive face is positive and on the negative face the negative direction of these is considered positive. This sign convention is in tune with the general sign convention mentioned previously.

In some books you may find one of the sign conventions for either shear force or bending moment will be same as shown in the figure and the other would be just the reverse. This may modify the final equation. Always make it a habit to show the sign convention adopted along with the SFD or BMD as shown above.

1. Spring Stiffness Test

Objective

The objective of this experiment is to find the spring constant for the given springs and to compare the values with the theoretical values.

Apparatus

Spring is a deformable member and it can support both tensile and compressive forces. Special springs are designed to support bending or torsional loads. In this experiment we confine our attention to the study of springs supporting tensile or compressive forces. Knowledge of the force-deformation characteristic of a spring is essential to design springs for an application.

The apparatus setup consists of a spring suspension system for the spring to be loaded in compression or tension to conduct the experiment (Fig. 1). A rod is provided to facilitate keeping the spring axial. Weights are available in different denominations for loading. Vernier scale is provided with the setup to measure the deformation (elongation /compression) of the spring.

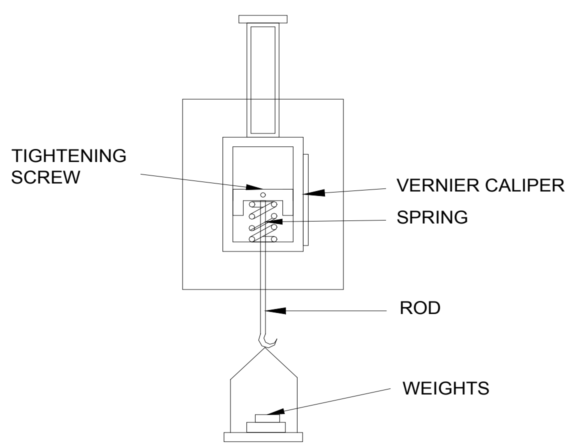


Fig 1 Spring Loading apparatus

Theory

Experimental determination of the force-deformation behaviour of a spring is simple and straight forward. If the force-deformation behaviour is linear then one has a linear spring. In such a case the amount of force required to produce a unit deflection is called the spring constant of the spring. In other words,

$$F = k\delta \tag{1}$$

Where F is the force in Newton, k is the spring constant or stiffness of the spring (N/mm) and δ is the spring deformation (elongation or compression in mm).

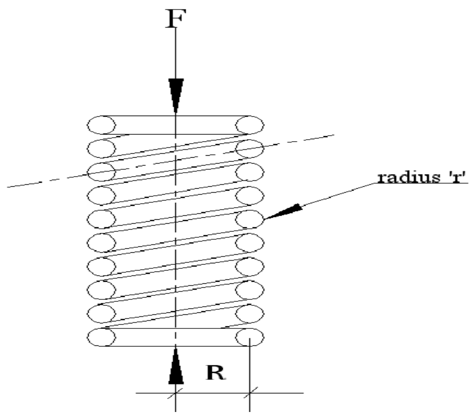


Fig. 2 Geometric and loading details of the spring

The spring constant of a spring can also be computed analytically. The geometric details of the spring and the elastic properties of the material of the spring are needed to compute this. However, the computational approach is not quite simple. When force is applied, spring as a whole is compressed or elongated but this overall deformation comes about due to torsional/bending deformation of the spring wire. Knowledge of torsion and bending is essential to understand the analytical procedure.

When the diameter of the wire is small in comparison with the radius of the coil (Fig.2), an element of the spring between two closely adjoining sections through the wire can be considered as a straight circular bar subjected to torsion for the external loading configuration as shown.The spring stiffness is given by

$$k = \frac{Gr^4}{4nR^3} \tag{2}$$

where G is the shear modulus of the spring material (GPa), n is the number of active coils, r is the spring wire radius (mm) & R is the mean radius (mm) of the spring.

Specimen

Three helical springs of different diameters.

Procedure

1. Measure the dimensions of the spring radius R and r (wire radius) using the vernier calipers.
2. Place the spring in the loading setup and then clamp it by tightening the screw.
3. Note down the vernier scale reading for no load condition.
4. Load the spring in steps of 0.5 kg of weight up to 2.5 kg, and note down the readings from the vernier scale.
5. Remove the loads one by one when the loading is over and note down the deflection when each load is removed.
6. Take the spring out of the setup and repeat experiment on the other two springs.
7. Calculate the spring stiffness using the above readings.

Sample Table

Spring No. : _____, Mean Radius (*R*):________, Free Length:________, Wire Radius:_________

Number of coils (*n*):_____________, Initial Reading of the vernier:_________, Least count of Vernier:______

S.No	Load		Loading		Unloading	
	kg	N	Vernier reading	Deflection	Vernier reading	Deflection

Analysis of Results

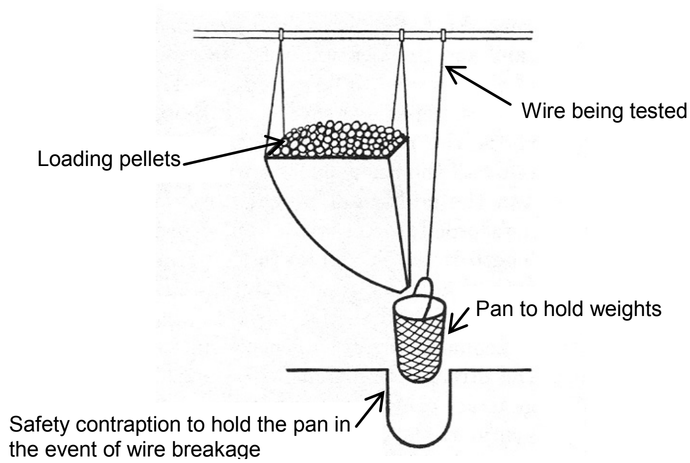
Draw a graph between the load and deflection of the spring for all the three cases. Compare the stiffness values calculated with the theoretical values for all the three springs. Tabulate you results indicating the percentage difference. Comment on reasons for deviation.

Points for Discussion

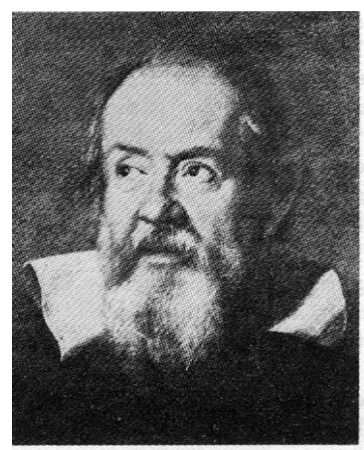
1. How does spring constant change with wire dia?
2. How does spring constant change with spring dia?
3. What would be the nature of force–deformation relationship for a spring of variable diameter?*
4. Comment about the stress distribution across the wire dia.*

Leonardo da Vinci (1452 – 1519)

Studied the strength of structural materials experimentally. Interestingly the early studies on materials were done on wires and he gives a method to find the load an iron wire can carry. The focus was confined to finding the maximum elongation and the failure load.



Galileo (1564 – 1642)



Attempts to find the safe dimensions of structural elements analytically were initiated by Galileo in the seventeenth century.

Initiated some studies on designing beams of constant strength. Also discussed strength of hollow beams and states that such beams "are employed in art-and still more often in nature – in a thousand operations for the purpose of greatly increasing strength without adding to weight; examples of these are seen in the bones of birds and in many kinds of reeds which are light and highly resistant both to bending and breaking…" Comparing a hollow cylinder to a solid one of the same cross-sectional area, Galileo rightly concluded that the hollow one has higher bending strength.

2. The Tension Test

Objective

The objective of this experiment is to determine the tensile behavior of the given specimen. Mechanical properties such as modulus of elasticity, yield strength, ultimate strength, and ductility are also to be determined for the given material.

Apparatus

The universal testing machine is used for testing the specimen. The testing machine applies tensile or compressive forces by means of moving crosshead. Vernier calipers are available to measure the dimensions of the specimen before testing. The machine is furnished with a proving ring load cell that facilitates the calculation of the applied load.A dial gauge is to be installed to measure the elongation of the specimen.

Genesis of the Theory of Stress and Strain

Determination of force-deformation relationship of a material is quite important. To determine the uniaxial load-elongation characteristics of a particular material one can take a rod of that specimen and apply loads at the ends and measure the deformation. Consider the deformation of three rods of identical material, but having different lengths and cross-sectional areas as shown (Fig.1).

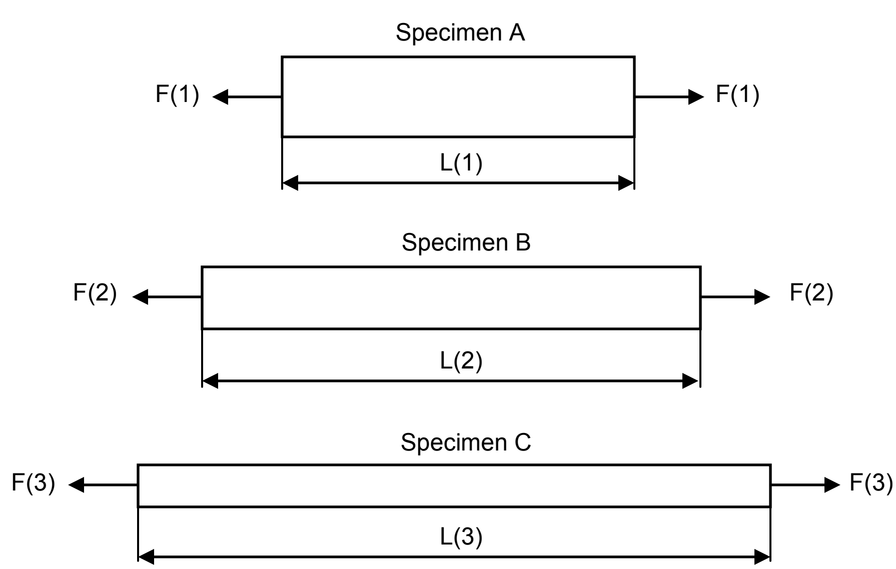


Fig. 1 Specimen dimensions

Assume that for each bar the load is gradually increased from zero and the elongation δ is measured. If the maximum elongation is very small (not greater than 1% of the original length) then for most materials the results of the three tests would be a plot of three separate lines as shown in Fig. 2. This leads to difficulties in characterizing the material as one needs to perform a separate test for each length and for each cross-section!

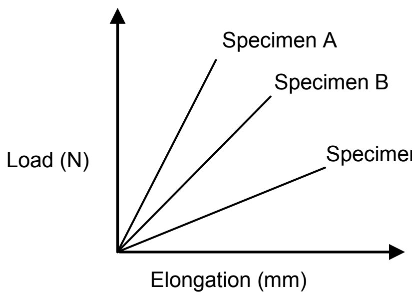


Fig. 2 Load - Deflection curves

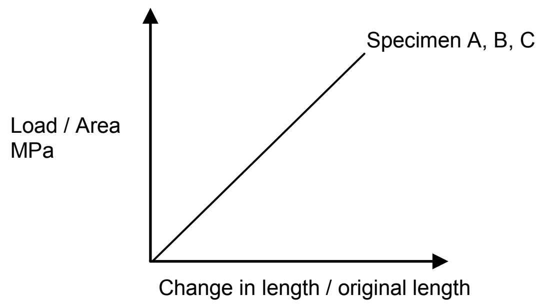


Fig. 3 Load / Area, vs
Change in length / original length

If the experimental data are re-plotted with load over area as ordinate and elongation over original length as abscissa, the test results for the three bars can be represented by a single curve as shown in Fig.3. This re-plotting simplifies the problem of determining the load deformation behavior of materials.

Thus a single test is sufficient to characterize the elastic behavior of the material. The slope of the force-deformation relationship is called the Modulus of elasticity and is denoted by the symbol E .

$$E = \frac{P/A}{\delta/L} = \text{Average stress / Average strain}$$

The above experimental approach introduced newer derived quantities namely stress and strain. Stress is force divided by area and strain is change in length by original length. This is a very elementary definition of stress and strain and they appear to be scalars. However, they are in fact **tensors** and would be developed in a course on Mechanics of Solids.

It is interesting to note that when the load-deflection curve is linear, a solid bar subjected to end loads act in the same manner as the coiled spring and the spring constant here is,

$$k = \frac{P}{\delta} = \frac{AE}{L}$$

The Tension Test

The strength of a material depends on its ability to sustain a load without undue deformation or failure. This property is inherent in the material itself and must be determined by experiments. As a result, several types of tests have been developed to evaluate a material's strength under loads that are static, cyclic, or impulsive. One of the important tests to perform is the tension test.

The tension test is used primarily to determine the relationship between the average normal stress and the normal strain in many engineering materials such as metals, ceramics, polymers, and composites. To perform this test, a specimen of the material is made into a "standard" shape and size, which is used as the basis for calculating the average strain.

Before testing, two small marks are identified along the specimen's length. The distance between the marks is termed as the gauge length of the specimen. These marks are located away from the ends of the specimen because the stress distribution at the ends is somewhat complex due to the gripping at the connections where the load is applied.

A typical stress-strain graph for the mild steel specimen, until it's fracture is shown in the following diagram (Fig. 4). You will be using a very simple specimen for the test. Detailed codes exist on how to make a specimen and how to interpret the test results. This is given in Appendix-1.

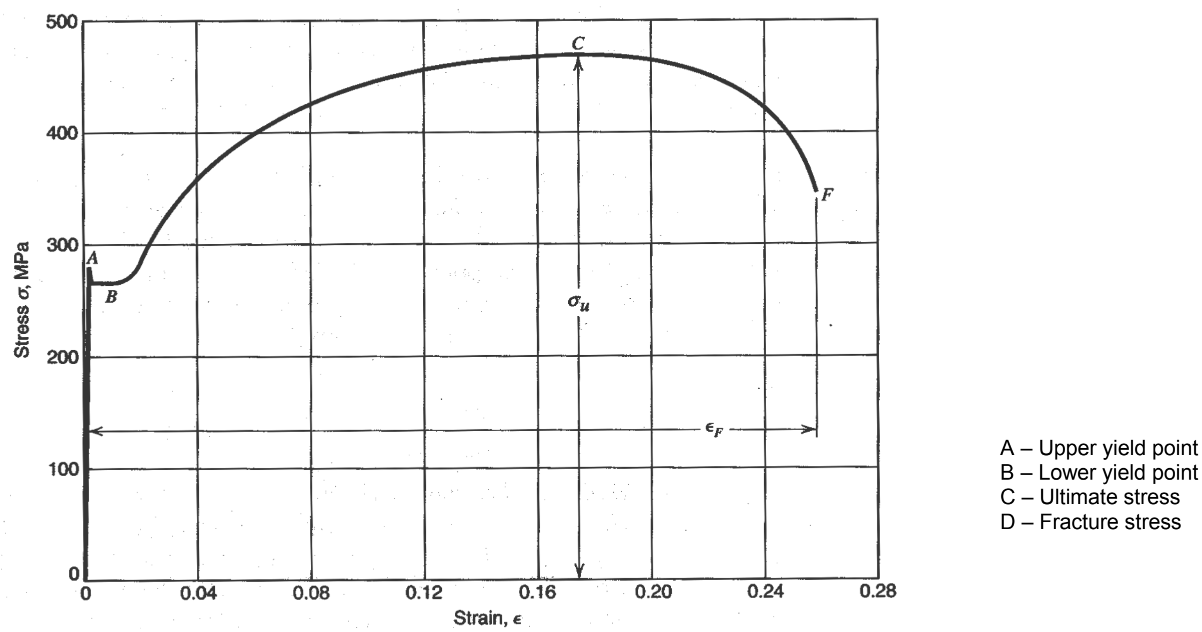


Fig. 4 Typical engineering stress-strain curve for mild steal (Note the value of strain at fracture)

The salient features of the stress-strain curve are defined next. The codes again deal with several subtleties in these definitions, which can be seen in Appendix-1.

Elastic behavior

The specimen is said to respond elastically if it returns to its original shape or length when the load acting on it is removed. In elastic region the stress is proportional to strain and the slope of the straight line in this region gives the Young's modulus *E*.

Young's modulus, $E = \frac{Stress}{Strain}$

The upper stress limit to this relationship is called the *proportional limit*. If the stress slightly exceeds the proportional limit, the material may still respond elastically, however the curve tends to be non-linear causing a greater increment of strain for corresponding increment of stress. This continues until the stress reaches the *elastic limit*.

Yielding

A slight increase in stress above the elastic limit will cause the specimen to deform permanently. This behavior is called yielding and the stress at which yielding begins is called the *yield stress* and the permanent deformation that occurs is called plastic deformation.

Strain hardening

A further load beyond the yield stress applied to the specimen, causes the force–deformation curve to rise continuously with decreasing slope until it reaches a maximum stress referred to as the ultimate stress. The rise in the curve in this manner is called *strain hardening***.** It is named such to indicate that even after the yielding; the specimen can resist loads and the resistance increases as a function of strain until ultimate stress is reached.

In most analytical models, to simplify computation, the stress-strain curve is assumed to be a horizontal line beyond yielding. Only in advanced research, the actual material behavior beyond yielding is satisfactorily modeled with considerable mathematics.

Necking

At the ultimate stress, the cross-sectional area begins to decrease in a localized region of the specimen, instead over its entire length. As a result, a constriction or "neck" gradually tends to form in this region as the specimen elongates. The tensile test reaches its conclusion when a small crack develops at the center of the neck and spreads outwards to complete fracture.

Procedure

1. For the given specimen, measure the mean diameter, taking the average of three measurements.
2. Set up the universal testing machine

   Install the specimen. Use the conical friction wedges to fix the specimen in the machine. The load should be nearly zero.
3. Attach the dial gauge to the experimental setup and make sure that the indicator displays zero.
4. Perform a final check of the entire setup.
5. Load the specimen by rotating the hand wheel, in the testing machine.
6. Note down the elongation of the specimen for each five divisions of the load applied.
7. Note down the yield point, ultimate point, by observing the significant changes in the indicator movement of the load measuring dial gauge.

8. During the test, carefully observe the specimen as it is being deformed, and note down any change in the shape.
9. After the specimen fails, remove the specimen pieces from the testing machine.
10. Make a detailed record of the general features of the fractured surfaces, and measure the diameter of the specimen at the point of failure.

Observations

1. Initial length of the portion of the specimen observed = __________ mm
2. Initial diameter of the portion of the specimen observed = __________ mm
3. Final length of the portion of the specimen under consideration= __________ mm
4. Neck diameter = __________ mm

S.No	Load Applied		Extension of the specimen		Stress	Strain
	No.of Div.	N	*No.of Div.*	mm	N/mm^2	

Analysis of Results

1. Plot the correlation between the engineering stress vs. engineering strain for the material tested.
2. From the stress-strain diagram of the material, compute:
   (a) Modulus of elasticity,
   (b) Yield strength,
   (c) Ultimate strength
   (Hint: for a ductile material without a clear yield point, the yield strength can be determined using a 0.2% offset strain)
3. Compute the percentage reduction in area % RA, and the percentage elongation %EL, for the given material using the following definitions.

$$\%RA = \frac{A_0 - A_f}{A_0} \text{x}100\% \quad \text{and} \quad \%EL = \frac{l_f - l_0}{l_0} \text{x}100\%$$

where, A_0 is Initial area of cross section of the specimen, A_f is Area of cross section at the point of necking, after failure, l_0 is Initial length of the specimen, l_f is Final length of the specimen.

Points for Discussion

1. What is the principle of the proving ring?
2. What is the difference between the elastic limit and the proportional limit?
3. What are the upper and lower yield points?
4. Why the breaking strength is less than the ultimate strength?
5. What is the point of instability?
6. Describe in detail the process by which the cup and cone type of failure occurs in the ductile material. Why this mode of failure is not observed in brittle materials?*
7. Compare the true and engineering stress-strain curves for the given material. Compute the true and engineering stresses at the maximum load, and comment on the differences between the two values*.
8. Sketch the stress-strain relations for*,
   (a) Rigid material
   (b) Linearly elastic material
   (c) Perfectly plastic material (no strain hardening)
   (d) Rigid-Plastic material (with strain hardening)
   (e) Elastic-perfectly plastic material
9. For a generic point on the rod draw the Mohr's circle of stress*.

3. Understanding of Failure Planes

Objective

To appreciate that the plane of failure is a function of loading and the nature of the material.

Theory

As a structural engineer one would like to design a structure such that it does not fail in normal operating loads or due to sudden over loads. The structural dimensions are estimated based on the realistic modeling of the loads acting on the structure and the choice of the material of the structure and its capacity to withstand it.

One of the worst modes of failure is the material separation. An understanding on how the structure would tend to resist the loads applied and also which is the weakest plane along which failure can take place is of importance. To simplify the understanding, in a first course on Mechanics of Solids one studies the response of slender members subjected to simple loading situations such as axial loading, bending or torsion. As the loading is changed, the response of the slender member to resist the external loading is also changed.

The plane on which failure (separation) can occur is dictated by the material of the slender member. The response is different if the slender member is made of a brittle or a ductile material.

Theoretically, the plane of failure can be predicted if the stress tensor is known at the point of interest and a model of failure theory exists for the material.

Experimental Observation

Apply axial pull to the chalk. Increase the load until it breaks. Observe and record the plane of failure. Compare it with what happened to the steel rod failed in the tensile test.

Apply torque to the chalk. Increase the torque until it breaks. Observe and record the plane of failure.

Apply three-point bending to the chalk. Increase the central load until it breaks. Observe and record the plane of failure.

Points for Discussion

1. For each case write the stress tensor on a general point along the plane of failure*
2. Does maximum principal stress theory helps to predict the failure plane in each case?*
3. What would be the plane of failure if maximum shear stress theory is applicable for the material?*

Robert Hooke (1635 – 1703)

In 1662, Hooke become curator of the experiments of the Royal Society and his knowledge of mechanics and inventive ability were put to good use by the Society. He was always ready to devise apparatus to demonstrate his own ideas or to illustrate and clarify any point arising in the discussion of the Fellows.

In 1678 the paper "De Potentiâ Restitutiva" or "of spring" was published and was the first paper in which the elastic properties of materials are discussed. The linear relation between the force and the deformation is called the Hooke's law which is the foundation on which the future of mechanics of elastic bodies is built.

Robert Hooke not only established the relation between the magnitude of forces and the deformations that they produce but also suggested several experiments in which this relation can be used in solving some very important problems.

Thomas Young (1773 –1829)

Young showed his unusual ability not only in solving purely scientific problems but also in attacking practical engineering difficulties.

Young contributed much to strength of materials by introducing the notion of a modulus in tension and compression. He was also the pioneer in analyzing stresses brought about by impact.

In 1801, Young made his famous discovery of the interference of light and in 1802 was elected a member of the Royal Society.

Young was a failure as a teacher for his presentation was usually too terse and he seemed unable to sense and dwell upon those parts of his subjects that were difficult to the students.

"A philosophical fact, a difficult calculation, an ingenious instrument, or a new invention, would engage his attention".

4. Torsion Test

Objective

The objective of this experiment is to verify the torsional formula, compare the torsional stiffness of solid and hollow shafts and to evaluate the shear modulus of the shaft material.

Apparatus

The apparatus (Fig.1) setup consists of fixtures for holding the specimen and is provided with the lever arm and weighing pans for loading the shaft in pure torsion. The telescope and scale arrangement is to measure the twist of the shaft. The lamp is used for clear vision of the readings in the telescope.

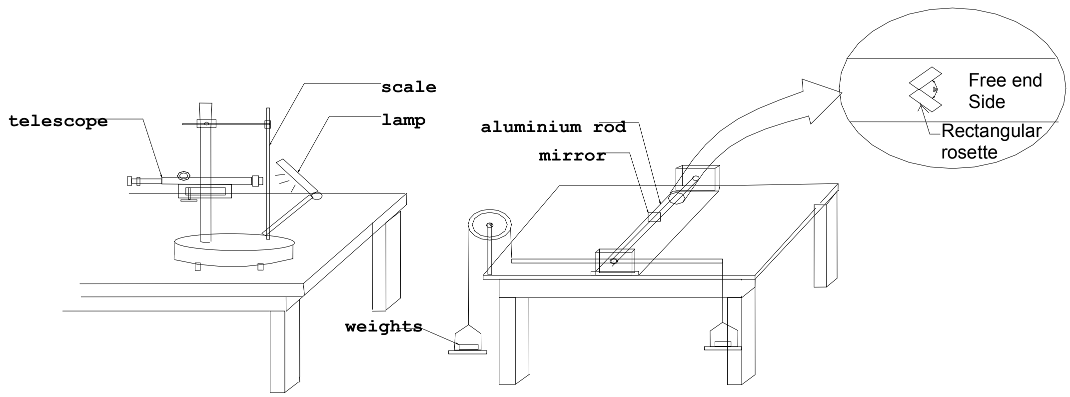


Fig. 1 Experimental setup

Theory

A slender member subjected primarily to twist is usually called a shaft. Shafts are used in the transfer of mechanical power from one point to another. In such an application, one is primarily interested in the twisting moment, which can be transmitted by the shaft without damage to the material. Knowledge of stresses that develop due to twisting (and its variation over the cross-section) is necessary to be known. In certain applications twisted shafts are used as a spring with prescribed stiffness with respect to rotation. In such a case, one is interested primarily in the relation between the applied twisting moment and the resulting angular twist of the shaft. For a circular shaft of constant diameter transmitting a uniform torque, the torsion formula is

$$\frac{M_t}{J} = \frac{\tau_{\theta z}}{r} = \frac{G\phi}{L} \tag{1}$$

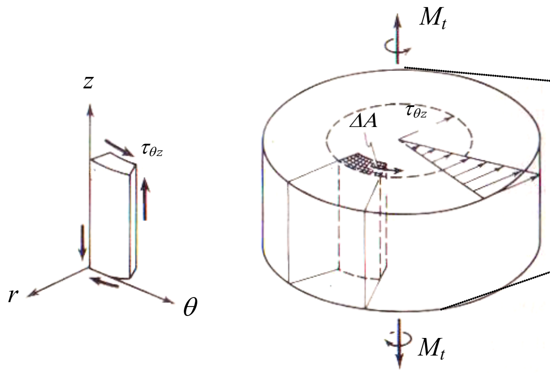


Fig. 2 Figure showing shear stress distribution

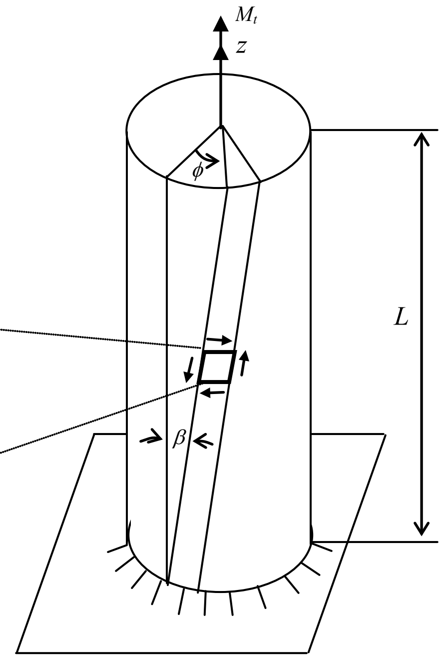


Fig. 3 Figure showing the definition of angle of twist

where, M_t is the twisting moment applied, J is the polar moment of inertia of the cross-section, $\tau_{\theta z}$ is the shear stress developed due to torsion (Fig. 2), r is the radius of the element being considered, G is shear modulus of the material, ϕ is angle of twist (Fig. 3) and L is the length of the uniform shaft.

Specimen

Solid and hollow Aluminium shafts

Procedure

1. Measure the cross sectional details of the shaft.
2. Fix the hollow aluminium shaft in the setup.
3. Adjust the telescope in such away that the image of the scale can be seen through the mirror on the shaft.
4. Measure the length of the shaft (L), distance from the fixed end to the center of the mirror (L_1), distance from the center of the mirror to the scale (L_2).
5. Note down the initial readings (x_1) of the telescope without applying any load.
6. Figure out how pure torsion is applied to the shaft. Apply a load of 200 g in the pan. Note: Loads should be applied simultaneously on both the weighing pans so that the setup does not get disturbed during loading.
7. Due to loading, the shaft is twisted. Note down the corresponding reading (x_2) by mirror and telescope arrangements.
8. Figure out how to find the angle of twist from these readings. (Hint: Since the scale is viewed through a mirror, the horizontal distance is twice the physical distance.) Verify your formula with the student representative before leaving the laboratory.
9. Load the shaft in steps of 200g until 1 kg and note down the readings as above.

10. Unload the weights in steps of 200g and record your readings.
11. Repeat the above procedure for solid aluminium shaft also.

Analysis of Results

1. Draw the free body diagram of the loaded specimen.
2. Plot a graph showing the torque versus angle of twist and torque versus maximum shear stress.

Sample Table

Shaft: Hollow shaft Dimensions: r_o = _______ , r_i =_______, L = ______,
Initial telescope reading (x_1) = ________ , Distance of mirror from scale (L_1) = _______

S.No	Load N	Torque N-m	Telescope readings (x_2)			(x_2 - x_1) mm	Angle of twist (radians)
			Loading mm	Unloading mm	Average mm		

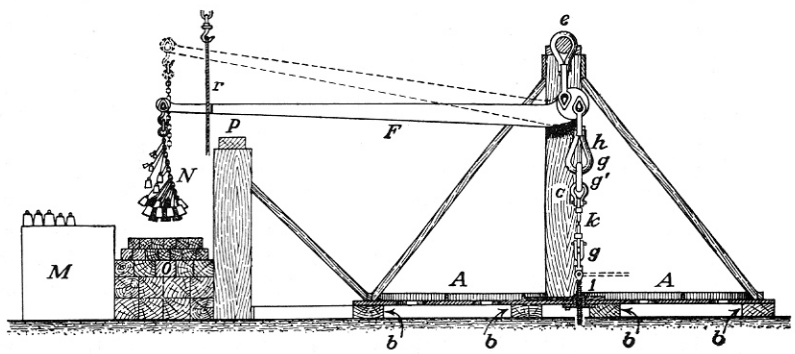
S.No	Torque Applied N-m	ϕ_{expt}	ϕ_{theory}	Percentage difference	G_{expt} (GPa)	G_{th} (GPa)	Percentage difference	Shear stress $\tau_{\theta z}$ MPa	Stiffness k_{expt}

For ϕ_{theory} use G for aluminium as =70GPa

Points for Discussion

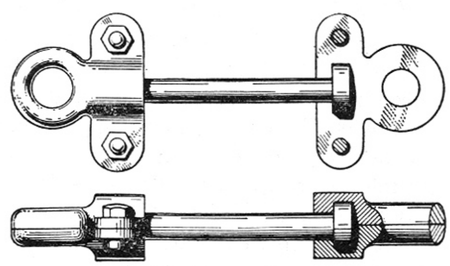
1. Does plane section remain plane before and after loading for a circular shaft subjected to torsion? Justify your answer by drawing a grid of vertical and horizontal lines over the shaft length.
2. Compare the stiffness and cross sectional areas of solid and hollow aluminum shaft. What do you infer from it?
3. How is shear modulus related to Young's modulus?*
4. For a generic point on the shaft surface draw the Mohr's circle of stress*

Kirkaldy conducted experiments between 1858 to 1861 and obtained the most complete description of mechanical properties of iron and steel then available. He was unable to measure elastic elongations or to calculate Young's modulus as instruments used by him were crude.



Kirkaldy's testing machine.

He was the first to recognize the effect of the shape of specimens on the ultimate strength. He has also understood the phenomenon of quenching better. He is an outstanding experimenter and has raised and explained many important questions in the field of mechanical testing of materials and his book is still of interest to engineers occupied with this subject.



Kirkaldy's tensile-test specimens.

Cauchy (1789 – 1857)

He is known for his formula to get stress vector on any arbitrary plane, correctly derived differential equations of equilibrium in three dimensions. He also established relations between six stress and strain components and initiated studies on torsion of prismatic bars which were later carried forward by Saint-Venant.

5. Measurement of Stress due to Bending Using a Strain Gage

Objective

Using Strain gages, find the stress on the top surface of a cantilever, near to fixed end, when loaded at free end.

Apparatus

Strain gage, Specimen bar (Al), Bar holder with provision for loading, Strain Indicator model P-3500, Multimeter.

Setup

The general setup is shown in Fig. 1. The strain gauged cantilever is attached to a 'holder' having a micrometer to give deflection in steps of 0.5mm. Leadwires from the strain gauge are attached to the P-3500 which then displays the strain.

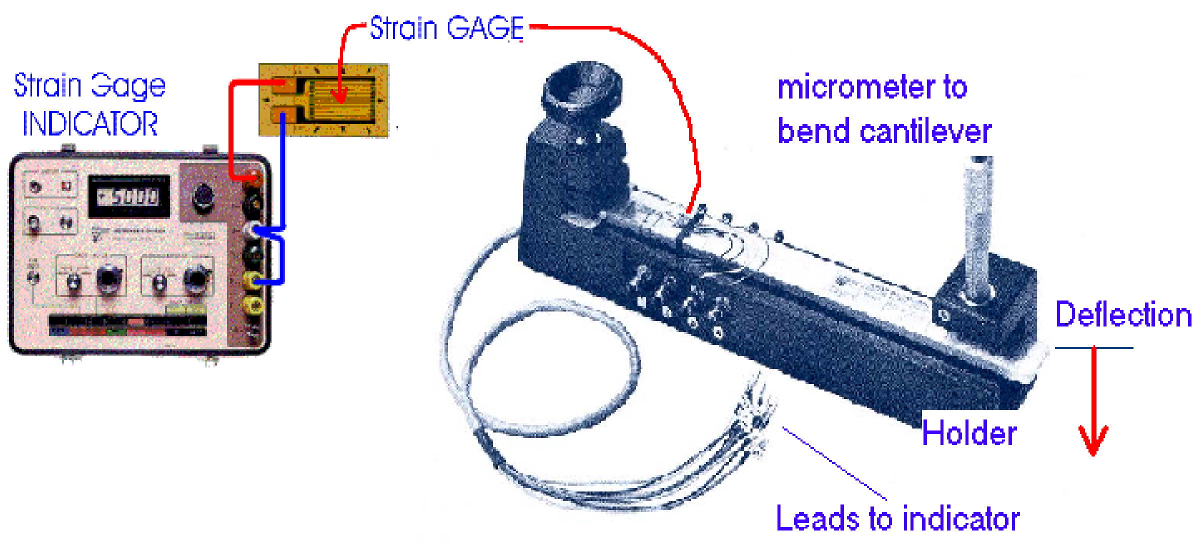


Fig. 1 General setup for experiment

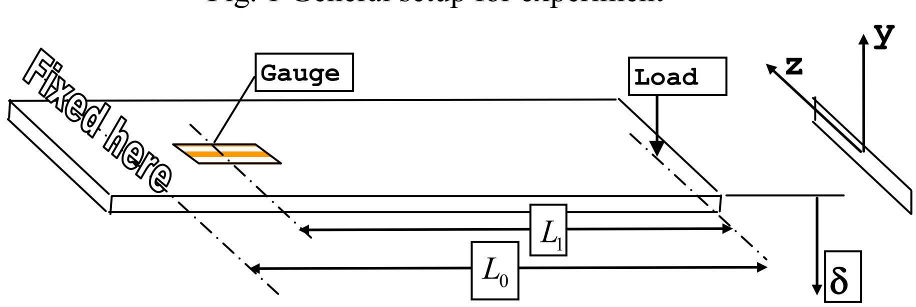


Fig. 2 Important dimensions and Co-ordinate system axes

Theory

Resistance strain gage is based on the phenomenon that the electrical resistance in a piece of wire is directly proportional to the length and inversely to the area of the cross section. If a resistance strain gage is properly attached onto the surface of a structure whose strain is to be measured, the strain gage wire/film will also elongate or contract with the structure, and as mentioned above, due to change in length and/or cross section, the resistance of the strain gage changes accordingly.

This change of resistance is measured using a strain indicator (with the Wheatstone bridge circuitry), and the strain is displayed by properly converting the change in resistance to strain. Every strain gage, by design, has a sensitivity factor called the gage factor which correlates strain and resistance as follows

$$\text{Gage factor } (S_G) = \frac{\Delta R / R}{\varepsilon}$$

Where R is resistance of un-deformed strain gage, ΔR is change in resistance of strain gage due to strain, and ε is strain.

Experimentally one measures strain. Stress can be computed by invoking the stress-strain relations. The stress-strain relations to the present case reduces to

$$\sigma_{xx} = E\varepsilon$$

The stress induced due to bending can be evaluated analytically using flexure formula. The flexure formula is

$$\frac{M_b}{I_{zz}} = -\frac{\sigma_{xx}}{y} = \frac{E}{\rho}$$

where M_b is the bending moment applied, I_{zz} is the moment of inertia of the beam cross-section, σ_{xx} is the normal stress acting on plane x in the direction x, y is the distance of the fiber from the centroidal axis, E is the Young's Modulus of the beam material and ρ is the radius of the curvature of the bent beam.

The beam is bent by applying a known deflection at free end. This can be converted to the free end load P by knowing the free end deflection analytically.

For a cantilever beam with end-load, the free-end deflection δ (in y direction) = $\dfrac{PL_0^3}{3EI_{zz}}$

$$\text{So, } P = \frac{3EI\delta}{L_0^3}$$

$$\text{Moment at gage location } M_t = -PL_1$$

Test procedure

1. Set the specimen bar (beam) to the bar holder so that the bar acts as a cantilever beam. Measure the important dimensions- L_0, L_1, breadth b, and thickness t.

The bar should not be loaded now, and for the following steps 2-7.

2. Measure the resistance of the strain gage using the multimeter and note it down

3. Connect the two ends of the strain gage as a QUARTER bridge as shown on the inner side of the strain indicator's lid (Fig. 3).

4. Depress the GAGE FACTOR button and set the (initial) gage factor to 2.005 or 2.06. *This value is supplied by the strain gauge manufacturer*. *Please refer the gauge spec sheet for this value*. Use the small four-position 'range selector' knob first and then the bigger potentiometer. Lock the potentiometer.

5. Depress the AMP ZERO button (amplifier)- the display should be +/-0000. else use the 'fingertip control' knob to bring +/- 0000.

6. Balance the circuit (still beam is not loaded). Depress the RUN button (with all other buttons OFF) and see the display. The present strain gauge actual output will be shown. Using the BALANCE knob, set the display to a convenient value (zero or any other value). Since the readings are going to be relative with respect to a point, it does not make any difference if the initial setting is zero or not as long as it is taken into account. If the initial setting is not zero, the initial value should be subtracted from the reading value. (You may have to use both the smaller and the bigger knobs). Lock the potentiometer.

7. With no load on cantilever, take the 1st set of readings. Note down the indicated strain.

8. For next step, make a deflection of 0.5mm with the micrometer handle. Add deflection in 0.5 mm steps, to a max of 5 mm. Repeat the measurements.

Note: Weight of the beam itself does contribute to the strain and may also be considered. However, since we zeroed instrument under the load of the beam weight it is irrelevant for our measurements.

Sample Table

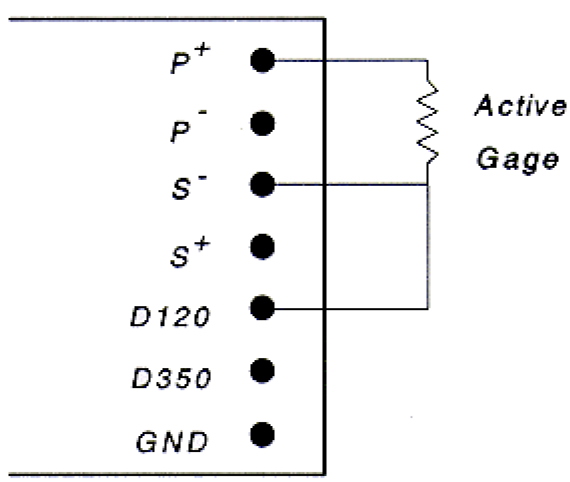
Initial reading of the strain indicator:________________

Sl no.	Deflection, mm	(□), micro strain		Strain (based on Beam theory)	% difference
		Display value	Strain (Experimental)		

Points for Discussion

1. Estimate the stress using stress-strain relations. Compare it with that predicted by beam theory.
2. If you have to consider the weight of the beam, how would you modify the experimental procedure?

3. If you have to account for the self-weight of the beam analytically how would you proceed?*



P-3500 Specification

Make : Measurements Group - Instruments division
Model : P 3500
Gage factor range: 0.5 to 9.99
Type of strain gages: 120 Ω and 350 Ω strain gages
Operation: Battery Operated
Readings: Displays strain as micro strain (□□).
Actual strain= display * 10e-6.
The instrument supports Quarter, Half and Full bridge circuits.
The display can be set to normal display (absolute value) or with a magnification of 10.

Fig. 3 Strain Measuring Bridge (instrument) Front panel, Quarter-bridge circuit connection details and P-3500 specifications.

6. Existence of Shear Stress in Beams

Objective

To demonstrate the existence of shear stress in a beam subjected to three-point bending (a simply supported beam subjected to a central load).

Theory

The famous flexure formula for predicting the bending behaviour of a slender member is developed with the premise that the bending moment acting on the beam does not vary along the length of the member. When the bending moment varies along the length of the member as in the case of three-point bending, cantilever with end load etc., shear stresses are developed to keep the beam in equilibrium.

The development of shear stresses can be demonstrated by the following simple experiment. One can also visualize the plane on which the shear stresses are developed by this experiment.

Experimental Observation

You are supplied with a plexiglass beam of 12mm depth and another plexiglass beam made of strips of 3mm thickness. Note that the surface of the beam strip is quite smooth and while you are doing the experiment ensure that the surfaces are not damaged in any way.

Place the 12mm depth beam on the supports and apply a central load of 0.5 kg. Measure the central deflection.

Now replace the beam by the set of four strips and as before apply a central load of 0.5 kg and measure the deflection. Also sketch carefully the deflected shape of each strip with particular care to the ends of the beam.

You are also supplied with four brass pins. Now insert these brass pins into the holes provides on the four strips – so that they act as one unit and repeat the above experiment and record your observations.

Points for Discussion

What do you infer from the observations you have made in the experiments?*

Could you identify the plane on which shear stresses develop?*

Instead of 3 mm strips if you are given twelve 1 mm strips what would be the deflection when pins are inserted?*

Does the presence of shear stress affect the assumption that plane sections before and after loading remain plane?*

In what class of problems is flexure formula exact even in the presence of shear stress?*

Johann Bauschinger (1833 – 1893)

He succeeded in getting a 100-ton machine for testing tensile properties of materials. Bauschinger invented a mirror extensometer to measure elongations of the order of 1×10^{-6}. With this instrument he could investigate the mechanical properties of materials with great accuracy. He noticed that a specimen which is stretched beyond the yield point does not show perfect elasticity and has a very low elastic limit under a second loading if it follows the initial stretching at once. But, if there is an interval of several days between the loadings, the material recovers its elastic property and exhibits a proportional limit that is somewhat higher than its initial value.

He examined the properties of mild steel when it is subjected to cyclic stress. He finds that if a specimen is stretched beyond its initial elastic limit in tension, its elastic limit in compression is lowered. Its value can be raised by subjecting the specimen to compression, but if this compression proceeds beyond a certain limit, the elastic limit in tension is lowered.

To render the experimental results of various laboratories comparable, Bauschinger arranged a conference in 1884 to arrive at uniformity in the procedures adopted. It was a grand success.

7. Deflection of Beams

Objective

To study the deflection of a simply supported beam and compare the experimental values of deflection with the theoretical values.

Equipments

Dial gauges, meter scale, vernier caliper, 500gm weights, weighing pan, aluminium and mild steel beams of rectangular cross section.

Theory

A beam is a slender member subjected to transverse loads. The beam bends to resist the loads applied. When transverse loads bend a beam, in general there will be both bending moment and shear force acting on each cross section of the beam. Both of these contribute to the deflection of the beam. When the beam deflection is measured experimentally one records the net effect.

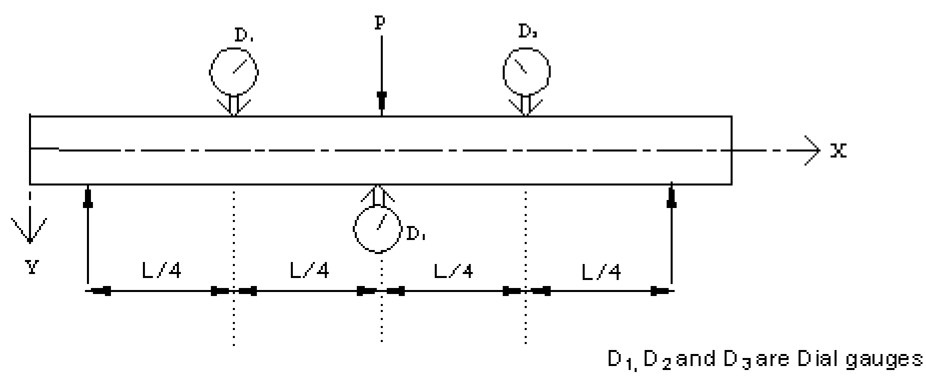


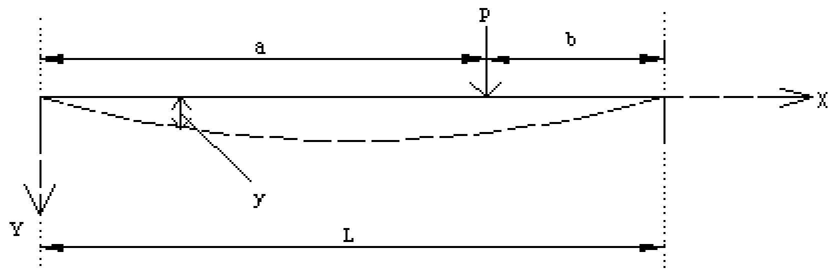
Fig.-1. Experimental apparatus for beam deflection

Analytical Expressions for Deflection

For analytical computations since the thickness of the beam is very small compared to other dimensions; the effect of shear forces is usually neglected. This is done to simplify the mathematics. It is well known that for a simply supported beam subjected to a load of P at the centre, the deflection at the centre neglecting the effects due to shear is

$$y_{\max.} = \frac{PL^3}{48EI} \tag{1}$$

Deflection at any point of a simply supported beam with the loading at an arbitrary position can be calculated using the formula.



$$y = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad (0 \le x \le a)$$

$$y = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) + \frac{P(x-a)^3}{6EI} \qquad (a \le x \le L) \qquad (2)$$

Where, E is Young's modulus, I is Second moment of area, y is Deflection, P is the Load applied and L is Length of the beam

Procedure

1. Place the beam over the supports at a known distance apart.
2. Place the dial gauges at the centre, on either side of the beam at a distance *'L/4'* from the center of the beam. Adjust the dial gauges such that the tip of the dial gauge remains in contact with the beam even when loaded.
3. The beam usually has an initial deflection due to its self-weight and the dial gauges readings are to be set to zero value.
4. Keep the loading pan exactly at the middle of the beam. Note down the weight of the loading pan. Place the weights in the weight pan such that it does not oscillate and note down the deflection of the beam by means of the dial gauges.
5. Load in steps of 500gm weight until it reaches 2500gm. For each loading, note the readings in the dial gauges.
6. Follow a similar procedure during unloading of the member and note the readings.
7. Now place the pan at '*L/4'* from one end and place the dial gauges under the load and at '*3L/4'* from one end of the beam.
8. Follow the above procedure for this case also and note the readings.
9. Now place the pan at *'3L/4'* from one end and place the dial gages under the load and at *'L/4'*.
10. Repeat the experiment for the aluminium specimen.

Sample Table

Location of loading: Specimen material: Specimen dimensions: Length, Thickness and Width

S. No.	Load applied		Deflection at (L/4)					Deflection at (3L/4)				
			Loading		Unloading		Avg. mm	Loading		Unloading		Avg. mm
	gm	N	div.	mm	div.	mm		div.	mm	div.	mm	

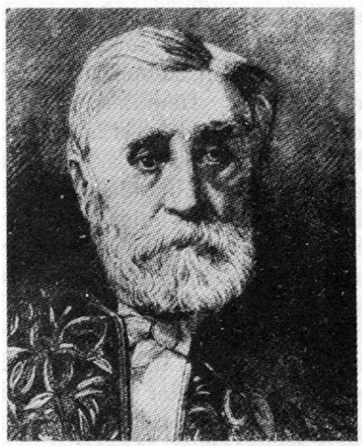
Deflection at load application point				
Loading		Unloading		Avg. mm
div.	mm	div.	mm	

Analysis of Results

1. Compare the experimental and theoretical values of deflection. Present it in the form of a graph.
2. Evaluate the stiffness of both the aluminium and mild steel beam. Comment on their comparison.
3. Discuss the reason for differences between the experimental and theoretical values of deflections.

Points for Discussion

1. Can you verify the validity of the formulae shown in Eq. (1) and (2)? Determine the percentage of deviation of the experimental results with theory. Tabulate your results.
2. Does the overhang in the beam affect the deflection measured?
3. Among, aluminium or mild steel specimen of same geometric dimensions which will deflect more?
4. Does moment of inertia affect the deflection?
5. Does self-weight of the beam play a role on the deflection? How will you modify the procedure to account for this in the experiment?
6. Is the deflection symmetric when the load is placed at the centre?
7. When the loading is asymmetric do you find any interesting relationship between the loading and the deflection? Comment*.

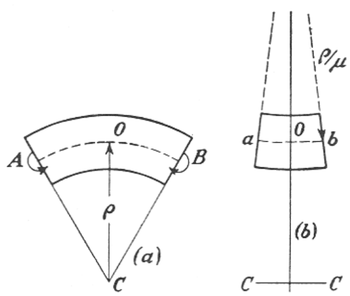


Barré de Saint-Venant (1797 – 1886)

Prominent Students Boussinesq, Lévy,

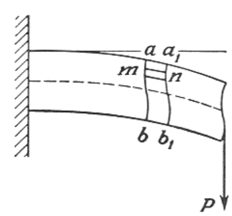
Saint-Venant believed that our knowledge can be improved only by combining experimental work with theoretical study. During the illness of Prof.Coriolis, Saint-Venant was asked to give lectures on Strength of Materials at the École de Polytechnic (France) and these lectures are of great historical interest, since some problems are mentioned which were later to become the objects of Timoshenko's scientific research. Saint-Venant was the first to try to bring the new developments in the theory of elasticity to the attention of his pupils.

Saint-Venant was the first to examine the accuracy of the fundamental assumptions regarding bending, viz., (1) that cross-sections of a beam remain plane during the deformation and (2) that the longitudinal fibres of a beam do not press upon each other during bending and are in a state of simple tension or compression.



He demonstrates that these two assumptions are rigorously fulfilled only in four-point bending. He also shows that the initially rectangular cross section changes its shape as shown in the figure having anticlastic curvatures due to lateral contraction of the fibres on the convex side and expansion on the concave side.

Saint-Venant was the first to study the role of shear stress in bending. He observed that for a cantilever loaded at free end, the cross-sections such as ab and a_1b_1 suffer warping as shown. Since, the warping is the same for any two cross-sections, it produces no changes in the length of the fibres and bending stress calculation by flexure formula is still exact.



8. Buckling of Struts

Objective

To study the buckling of a column under axially applied load with various boundary conditions and compare the experimental buckling loads with the Euler buckling formulae.

Theory

Compressive members can be seen in many structures. They can form part of a framework for instance in a roof truss, or they can be stand-alone as columns. Unlike a tension member which will generally separate into two if the ultimate tensile stress is exceeded, a compressive member can fail in two ways. The first is via rupture due to the direct stress, and the second by an elastic mode of failure called buckling.

Generally, short wide members that tend to fail by material crushing are called columns. Long thin compressive members that tend to fail by buckling are called struts. When buckling occurs, the strut will no longer carry any more load and it will simply continue to displace i.e., its stiffness then becomes zero and is useless as a structural member.

In this experiment you will load a strut until it buckles. One of the key points in the experiment is to determine precisely when the strut has buckled. The influences of various end conditions and the length of the strut in determining the buckling load is to be studied. To predict the buckling load analytically, use the Euler buckling formulae. The buckling load is also influenced by the imperfections in the strut.

Apparatus

Figure 1 shows the Buckling of struts experiment assembled in the Structures Test Frame. It has a digital force display. Make sure the Digital Force Display is 'on'. Check that the mini DIN lead from 'Force Input 1' on the Digital Force Display to the socket marked 'Force Output' on the right-hand side of the unit.

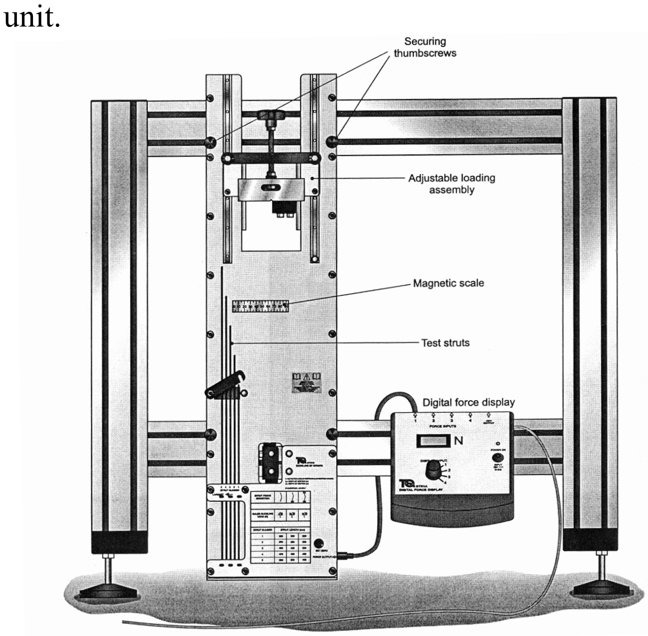


Fig. 1 Buckling of struts experiment in the structures frame

Carefully zero the force meter using the dial on the front panel of the instrument. Gently apply a small load with a finger to the top of the load cell mechanism and release. Zero the meter again if necessary. Repeat to ensure the meter returns to the zero.

Note: If the meter is only ± 1 N, lightly tap the frame (there may be a little 'stiction' and this should overcome it).

Figure 2 shows the details of the Buckling of Struts apparatus. It consists of a back plate with a load cell at one end and a device to load the struts at the top. The bottom chuck fixes to an articulated parallelogram mechanism, which prevents rotation but allows movement in the vertical direction against the ring load cell. The mechanism reacts to the considerable side thrust produced by the strut under buckling conditions, with little friction in the vertical direction.

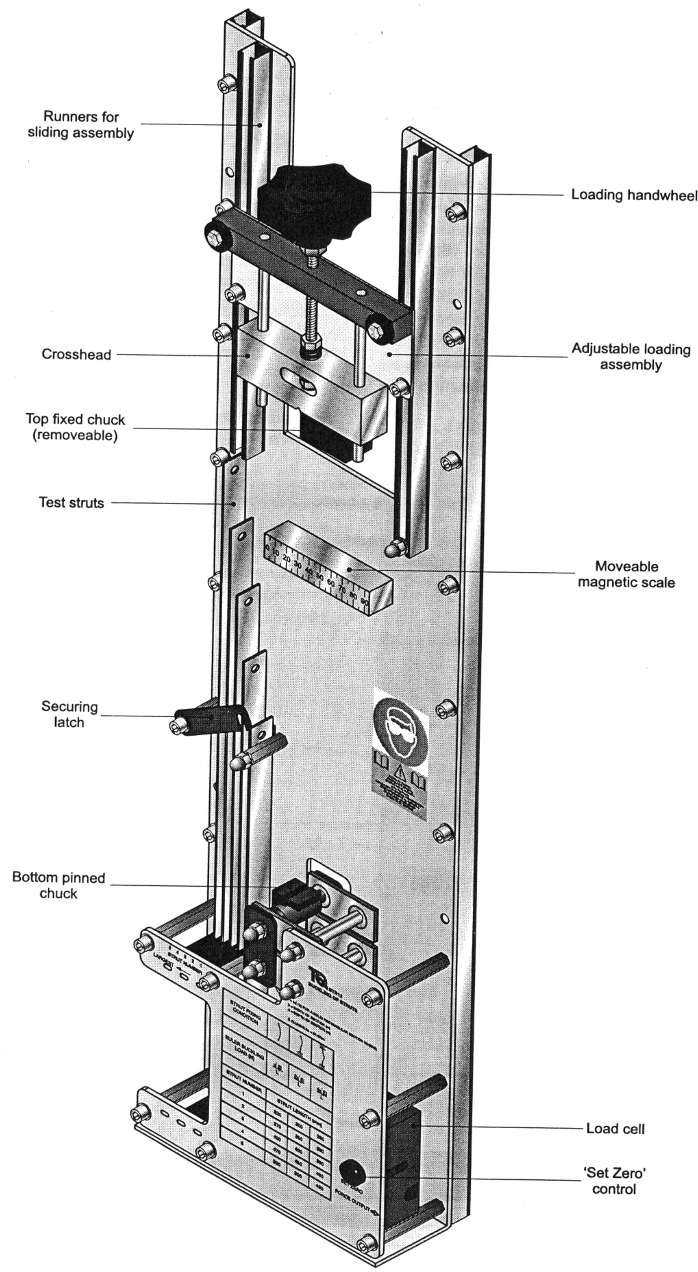


Fig. 2 Buckling of struts apparatus

Procedure

1. Buckling Load of a Pinned-Pinned End Strut

- Referring to Figure 3, fit the bottom chuck to the machine and remove the top chuck.
- Use the strut given for your experiments. Measure the cross section using the vernier.
- Adjust the position of the sliding cross crosshead to accept the strut using the thumbnuts to lock off the slider. Ensure that there is maximum amount of travel available on the hand wheel thread to compress the strut. Finally tighten the locking screws.
- Carefully back off the hand wheel so that the strut is resting in the notch but not transmitting any load; rezero the forcemeter using the front panel control.
- Carefully start the loading of the strut. If the strut begins to buckle to the left, “flick” the strut to the right and vice versa (this reduces any errors associated with the straightness of the strut).
- Turn the hand wheel until there is no further increase in load (the load may peak and then drop as it settles into the notches). **Do not load the struts after the buckling load has been reached otherwise the strut will become permanently deformed.** Record the final load as shown in sample table under ‘buckling load’. Try loading the strut three times and take the average of these as the experimental buckling load.

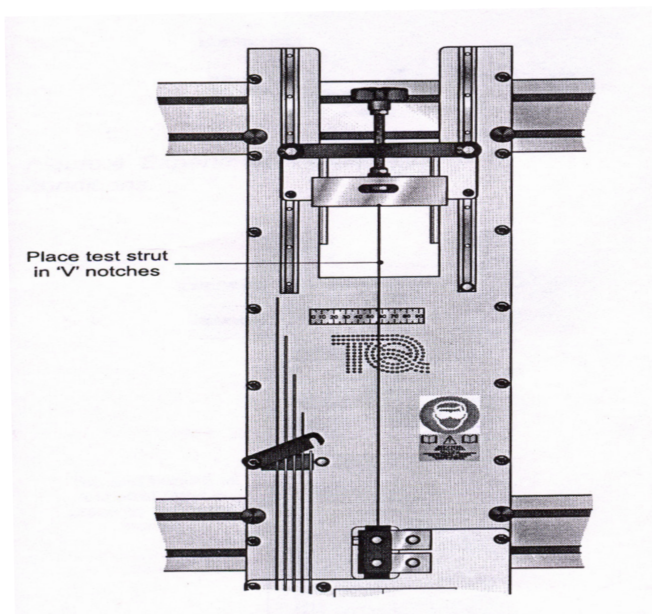


Figure 3 Experimental layout (Pinned-Pinned ends)

2. Buckling Load of a Pinned-Fixed End Strut

- Follow the same procedure as in Experiment 1, but this time remove the bottom chuck and clamp the specimen using the cap head screw and plate to make a pinned-fixed end condition (Fig. 4). **Do not load the struts after the buckling load has been reached otherwise the strut will become permanently deformed.**

- Record your results as per the sample table. Note that the test length of the struts is shorter than in Experiment 1 due to allowance made for clamping the specimen. Use this changed length in calculations.

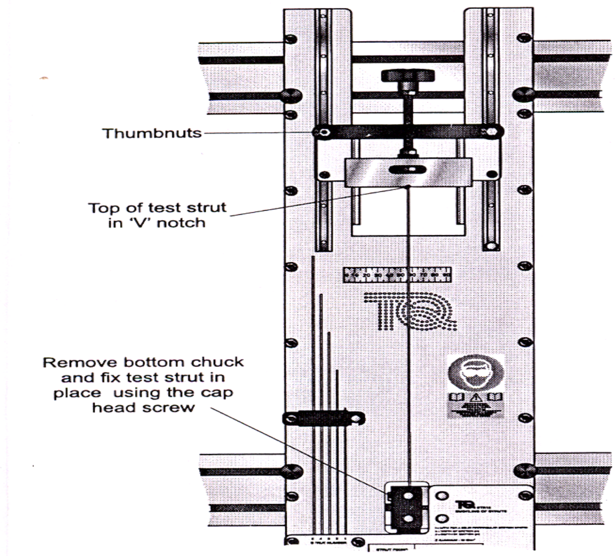


Fig. 4 Experimental layout (Fixed-pinned ends)

3. Buckling Load of a Fixed-Fixed End Strut

- Now fit the top chuck with the two cap head screws (Fig. 5) and clamp both ends of specimen, again this will reduce the experimental length of the specimen.
- Record your results as per sample table. Take extra care when loading the shorter struts near to the buckling loads. **Do not load the struts after the buckling load has been reached otherwise the strut will become permanently deformed.**

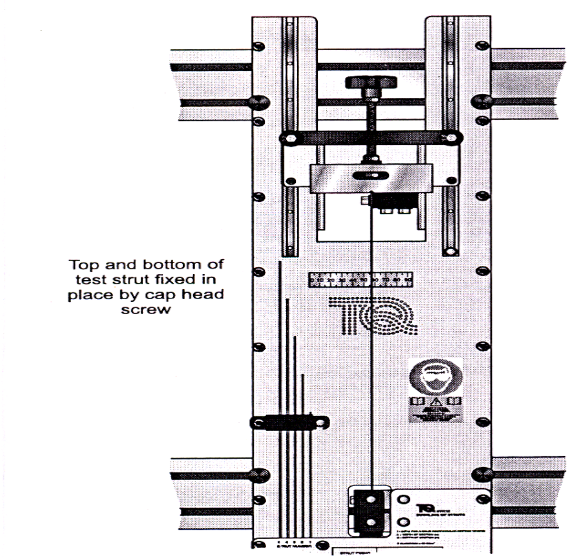


Fig. 5 Experimental layout (Fixed-Fixed ends)

Experimental Results and Analysis

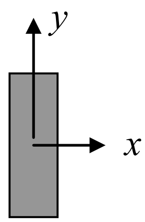
The general Euler buckling formula for struts

$$P_{cr} = \frac{\pi^2 EI}{L_e^{\,2}}$$

Where, P_{cr} is Euler buckling load (N), E is Young's modulus (GPa), I is Second moment of area (mm^4) Choose appropriately I_{xx} or I_{yy} for given cross section) L_e = Effective length of strut (mm);

(For Hinged-Hinged $L_e = L$, Hinged-Fixed = $\dfrac{L}{\sqrt{2}}$ and Fixed-Fixed = $\dfrac{L}{2}$)

Sample Table



End condition :

Strut No.	Length (L) mm	Thick-ness (d) mm	Breadth (b) mm	I_{xx} mm^4	I_{yy} mm^4	Buckling Load (P_{cr}) (Experimental) N		Buckling Load (Theory)	% Error
						Trial	Average		
1									

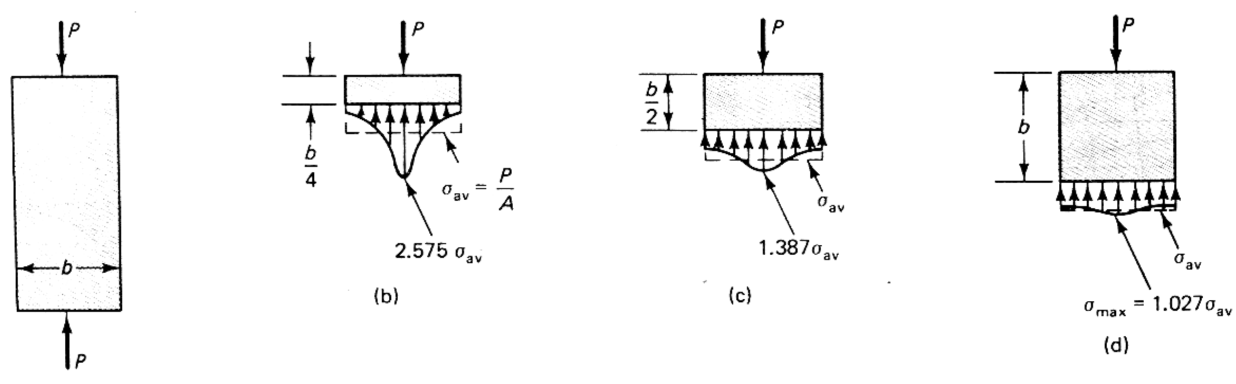
Points for Discussion

1. Draw the mode shapes for the buckling of the column for the boundary condition given in the experiment.
2. Tubular sections are more economical than solid sections for compression members. Why?*
3. Does the strength of the long column depend on the strength of its material?Explain why?*
4. List out the reasons for deviation of your results from the Euler buckling formula.

Saint Venant's Principle

Saint-Venant illustrated the principle which goes by his name by experimenting on rubber bars and showed that if a system of self-equilibrating forces is distributed on a small portion of the surface of the bar, a substantial deformation will be produced only in the vicinity of these forces.

In nutshell the principle states that for the purposes of analysis the actual loading can be replaced by a statically equivalent system. The disturbance thus introduced is local and its effect diminishes at distances away from the point of loading. This could be better understood by the following.



Consider that a uniformly distributed load needs to be applied to a bar as shown. Instead a concentrated load which is statically equivalent is applied at the end. The rigorous solution to this problem by theory of elasticity reveals that the loading becomes uniform at a distance *b*, which is the thickness of the specimen. Closer to the loading one cannot estimate the stress by invoking *P*/*A*. This can be done only beyond *b*.

This principle finds wide application in the mathematical theory of elasticity and in experimental work to design loading rigs.

In 1853, Saint-Venant presented his epoch-making memoir on torsion to the French Academic composed of Cauchy, Poncelet, Piobert and Lamé.

9. Study of Slender Members through Computer Animations

Objective

To study the response of slender members to various loading through animations based on Strength of Materials.

Computer Simulation

Based on Strength of Materials theory you are supplied with animations for the following.

Site address: http:\\apm.iitm.ac.in\smlab

1. Tensile test – Focus is on appreciating the various segments of the tensile test data.

2. Torsion of Circular cross-section. The twisting of the member as the torque is increased is presented. The animation is useful in appreciating the nature of deformation physically.

3. The response of a simply supported beam in supporting a central load is provided. Once can observe the deflection as well as generation of bending stress on a typical cross-section.

4. A pinned-pinned column subjected to buckling is presented. In the buckling experiment you will be finding the fundamental critical load. However, you will find the column has higher modes too. This is illustrated conveniently in this animation. An experimental verification of this needs skill.

Points for Discussion

In each case sketch your observations neatly. Pause the animation at least three times and sketch the deformation. The sketches thus prepared should give you a clear idea of how a slender member deforms when a particular loading is applied.

Franz Neuman (1798 – 1895)

Famous for initiating student seminars.

Prominent students : Borchardt, Clebsch, Kirchoff, Saalschütz, and Voigt.

He derived for the first time a formula for calculating the modulus in tension for the material of a prism cut out from a crystal with any arbitrary orientation. The most important contributions made by Neumann to the theory of elasticity are included in his great memoir dealing with double refraction – stepping stone to establish photoelasticity.

He has derived for the general case of a three-dimensional stress distribution. Using simple tests he showed how optical constants can be obtained and has applied his theory to the study of the coloured patterns which Brewester observed in the non uniformingly heated glass plates and pointed out that the double refractive property of such a plate is due to stresses produced by the nonuniform stress distribution.

Neumann was the first to study residual stresses.

Voigt (1850 – 1919)

His work finally settled the old controversy over the rariconstant and multiconstant theories. The questions were these : "Is elastic isotropy to be defined by one or two constants and, in the genral case, is elastic aeolotropy to be defined by 15 or 21 constants"? Voigt used thin prisms cut out from single crystals in various directions in his experiments. The elastic moduli were determined from torsional and bending tests of these prisms. In addition, the compressibility of crystals under uniform hydrostatic pressure was studied. The results disproved rariconstant theory and noted that one needs 2 constants for isotropy and 21 constants for aeolotropy. The result of his work also helped in coining the term tensor! (as he dealt with tensions of crystals of various planes).

10. Qualitative Visualization of Stress Fields by Photoelasticity

Objective

To learn the various polariscope arrangements and to distinguish between isochromatics and isoclinic contours.

Plane Polariscope Setup

1. Switch on the lamp house and swing the green filter out of the line of view.
2. Place the polarizer at the back of the rear element frame and the analyzer at the front of the front element frame.
3. Slowly rotate the analyzer until the light output from it is a minimum. The polarizing elements are now crossed i.e., their polarizing axes are at right angles to one another. This arrangement constitutes a plane polariscope.
4. Rotate each element until the arrow heads align with the zero's on the respective scales.

Circular Polariscope Setup

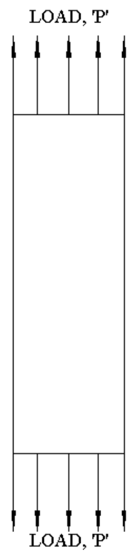
1. Set up a plane polariscope as described earlier.
2. Place quarter wave plate 1 on the front of the rear element frame.
3. Rotate this quarter wave plate to give a minimum light output as viewed through the analyzer.
4. Place the quarter wave plate 2 at the rear end of the front element frame and again rotate to give minimum light output.
5. With reference to the degree scales, rotate each quarter wave plate through 45°.
6. Keep the analyzer at horizontal and flip it to vertical. Record your observation and the background light.

Visualization of Stress Fields

Spanner and Nut

1. View the model in plane polariscope with white light as illumination.
2. Tighten the epoxy nut by the epoxy spanner.
3. Observe the formation of fringes. Lightly increase the tightening of the nut, record how the fringes change.
4. Distinguish between the zeroth order fringe and an isoclinic.
5. Verify your result by viewing the model in circular polariscope dark field.
6. Could you identify a correlation between the load applied and the formation of fringes?

Tension Strip under Uniform Loading



1. The tension strip made of a bi-refringent material is loaded as shown in figure.
2. Use a dark field circular polariscope with white light illumination to view the model.
3. Gradually increase the load.
4. Record your observations of color zones in the model.
5. Comment on the nature of stress field based on your experimental observations.

Fig.1 Tension Strip under axial loading

11. Evaluation of Stress Field In a Beam Under Four Point Bending by Photoelasticity

Objective

To study the whole field nature of the stress field in a beam under four point bending and to find the normal stress variation over the depth of the beam.

Equipment Required

Rectangular beam made of birefringent material, Loading Equipment.

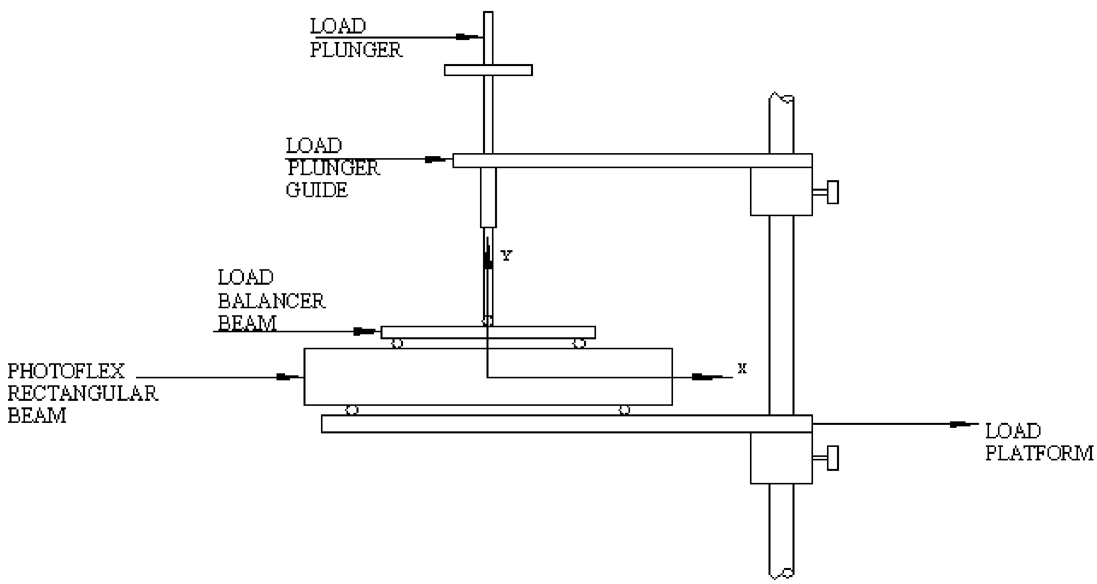


Fig.1 Loading Arrangement of the beam

Theory

Unlike other experiments you have done in this laboratory, you will see for the first time the visualization of field nature of stress in this experiment. Photoelasticity uses light as a sensor and the input and exit lights are analysed to evaluate the stress field in the model. Photoelasticity directly gives only the contours of difference in principal stresses (σ_1-σ_2) and its orientation θ. Usually a circular polariscope is used to determine the fringe order N corresponding to contours of σ_1-σ_2. Using stress optic law, the fringe order is interpreted as difference in principal stresses as,

$$\sigma_1 - \sigma_2 = \frac{NF_\sigma}{t} \tag{1}$$

where, N is the fringe order, F_σ is the material stress fringe value and t is the model thickness. For the beam material you use in the laboratory take F_σ as 0.3 N/mm/Fringe. Usually, this value is evaluated by conducting a separate experiment.

The stress field in the case of a beam under pure bending is quite simple and the flexure formula states

$$\frac{M_b}{I_{zz}} = -\frac{\sigma_{xx}}{y} = \frac{E}{\rho} \tag{2}$$

where M_b is the bending moment applied, I_{zz} is the moment of inertia of the beam cross-section, σ_{xx} is the normal stress acting on plane x in the direction x, y is the distance of the fiber from the centroidal axis, E is the Young's Modulus of the beam material and ρ is the radius of curvature of the bent beam.

Please note from Eq. (2) that σ_{xx} is a function of y but is independent of x. Theoretical modeling reveals that σ_{xx} varies linearly over the depth of the beam and this needs to be verified experimentally which is the purpose of this experiment.

Stress is a tensor and can be expressed with respect to any three mutually perpendicular directions as

$$\underset{\text{(a)}}{\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}} \quad \text{or} \quad \underset{\text{(b)}}{\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}} \tag{3}$$

Please note that the above three mutually perpendicular directions are different for case(a) and (b). Mathematical procedures exist on how to get (b) if (a) is given and vice-versa. This you will learn as part of the course on Strength of Materials / Mechanics of Solids.

In the case of a slender beam in pure bending, the only stress component that exists is σ_{xx} and the stress tensor is

$$\begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

Using the symbolism used in Eq. 3(b) and combining Eq. (1) and Eq. (4) one gets

$$\sigma_x = \frac{NF_\sigma}{t} \tag{5}$$

Thus, the experimental evaluation of normal stress variation over the depth of the beam reduces to the evaluation of fringe order variation over the depth of the beam.

Procedure

1. Set up the birefringent model to apply pure bending in the central section. Pure bending means bending without shear and this occurs in a region where the bending moment is constant.
2. Record the relevant distances to evaluate the bending moment acting on the beam.
3. Set the polariscope in dark field with circularly polarized light impinging on the model.
4. Apply a load of 200g and view the model in white light.
5. Identify the zeroth order fringe.
6. Identify the fringe order gradient in the central section of the beam. Use the Color Code displayed in the lab.
7. Insert the green filter and label the fringe orders. Measure the distance of the fringes from the bottom surface of the beam.
8. Sketch in your note book, the isochromatic fringe pattern and label its numbers.
9. Using stress optic law determine the distribution of normal stress over the depth of the beam and compare it with theory. (Use F_σ as 0.3 N/mm/fringe)
10. Repeat the experiment for loads of 400g and 600g.

Sample Table

Bending Moment(N-m)=

Load Applied = *N* F_σ = 0.3 N/mm/Fringe

Fringe Order *N*	Distance of fringe from Bottom Surface (mm)	Distance from Middle Surface, *y* (mm)	Normal Stress σ_{xx} (MPa)

Points for Discussion

1. Is the stress field symmetric about the neutral axis?
2. On the sketch of fringe patterns drawn for the complete beam, mark the zone of validity of flexure formula. Is experimental data away from this zone reliable?
3. The experiment is conducted on plastics. How do you correlate the results to metallic specimen?*
4. Does Poisson's ratio of the beam material has any influence on the formation of fringes? (*Hint*: Fringe order is directly proportional to the thickness. Study how does beam bending alters the thickness.)*

Instrument Characteristics – Terminologies

Readability of an instrument indicates the closeness with which the scale of the instrument may be read. The *least count is* the smallest difference between two indications that can be detected on the instrument scale. Both of these depend on scale length, spacing of graduations and parallax effects.

For an instrument with a digital readout, the terms readability and least count have little meaning.

Sensitivity of an instrument is the ratio of the linear movement of the pointer on an analog instrument to the change in the measured variable causing this motion. For example, a 1-mV recorder has a scale length of 10 cm then its sensitivity is 10cm/mV, assuming that the measurement is linear over the entire scale.

The term sensitivity does not have the same meaning for a digital instrument because different scale factors can be applied with the push of a button. Usually the manufacturer specifies the sensitivity for a certain scale setting, e.g., 100 nA on a 100 -μA scale range for current measurement.

Accuracy of an instrument indicates the deviation of the reading from a known input. It is expressed as a percentage of the full scale reading. Accuracy of the instrument needs to be monitored/improved by periodic calibration.

Precision of an instrument indicates its ability to reproduce a certain reading with a given accuracy. In actual usage, one does not know the value of the input. One is normally advised to make a few readings and the average of these is taken as the measured value. The deviation of the actual reading over this average is the precision of the instrument.

Consider that the five readings of a voltmeter reads the values 103, 105, 103, 105 and 104 V. The average value of this is 104 V and the maximum deviation is only 1 V. The precision of the equipment is then $\pm$ 1 percent. Suppose that this measurement was done for a known input of 100 V (from a well calibrated standard source) then the deviation of the instrument is high from the actual value and the maximum deviation is 5 V. Hence, the accuracy of this instrument is poor and the readings are not better than 5 percent. However, the accuracy can be made very close to the precision of the instrument by resorting to re-calibration of the instrument. Accuracy cannot be made better than the precision of the instrument. Precision of the instrument depends on various factors.

Hysteresis exists for an instrument if there is a difference in readings depending on whether the value of the measured quantity is approached from above or below.

The deviation of the actual reading from the input is the error in measurement. This error could be *systematic* or *random*. Systematic errors could be introduced due to faulty calibration of the instrument. Every effort must be made to eliminate the influence of systematic errors in measurement. Random errors – as the name indicates are truly random and every effort must be made to reduce this error. This could be due to error of parallax, poor reading of the chart and so on. Statistical methods of data analysis could reduce the influence of random errors in data interpretation.

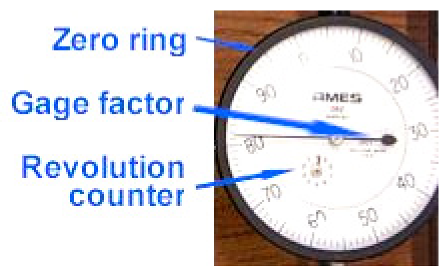
Measurement of Deflection – Dial Gauge

Dial Gages are used to accurately measure movements of tests in progress. They come in varying lengths and accuracies, so be sure to note gage factors and length.

The gage shown in the figure has a smallest increment of one thousandth of an inch (0.001), and can move 1 inch. Precision is listed as +/- 0.0005 inches.

Method to Use

Start by fastening the magnetic gage base to the equipment - simply push the button on the base in to activate the magnet.



- Adjust the holding assembly to place the tip against the object to be measured.
- Zero the gage by turning the black outside (zero) ring. Loosen locking nut first if it has one.
- As the test progresses, read the dial units. There are 100 units per revolution, so keep track of the revolutions counter as well.
- Always work in dial units and convert using the gage factor after testing. DO NOT CONVERT ON THE FLY!
- Multiply the dial units × gage factor (0.001) to calculate displacement in inches.

In this example, dial units = 41

History of Buckling of Columns

In experimental work with columns, inadequate attention had been paid to the end conditions, to the accuracy with which the load was applied, and to the elastic properties of material. Hence, the results of tests did not agree with theory and engineers preferred to use various empirical formulas in their designs.

In the beginning of the nineteenth century Duleau showed that if end conditions assumed in theory are fulfilled in experiments, Euler's formula gives satisfactory results for slender columns. Struts used in trusses are not slender and for these Euler formula gives exaggerated values of the buckling load. Towards finding a reliable formula recourse had to be had to experimentation.

In the middle of the nineteenth century the formulas that were derived on the basis of Hodgkinson's tests were widely used. In most of his tests, the bars had flat ends and the end conditions were somewhat indefinite. Sometimes, rounded ends were used, but the surfaces of contact were not spherical so that any buckling introduced some eccentricity. As a result, his formulas left something to be desired. Further, they were too complicated to be applied for practical problems.

The first reliable tests on columns were made by Bauschinger. By using conical attachments for his specimens, he ensured free rotation of the ends and central application of the load. The results thus obtained for slender bars agreed closely with Euler's formula.

Tetmajer continued the work of Bauschinger and many tests on steel columns were made at Germany. The tests showed that Euler's formula should be used for structural steel in calculating critical stresses when the slenderness ratio is larger than 110. For shorter specimens, a linear formula was offered which found wide acceptance in Europe.

In the book "An essay on the development of the theory of column buckling" by Jasinsky (1856-1893) one finds a correlation of theoretical and experimental investigations on columns.

Measurement of Force - Proving Ring

A proving ring is a device used to measure force. It consists of an elastic ring in which the deflection of the ring when loaded along a diameter is usually measured by means of a micrometer screw and a vibrating reed. The proving ring you will use in this laboratory uses a dial gauge to measure the change in diameter. The proving ring was developed by H. L. Whittemore and S. N. Petrenko of the National Institute of Standards and Technology (NIST), in 1927.

Proving ring consists of an elastic ring of known diameter with a measuring device located in the center of the ring. Proving rings come in a variety of sizes. They are made of a steel alloy. Manufacturing consists of rough machining from annealed forgings, heat treatement, and precision grinding to final size and finish.

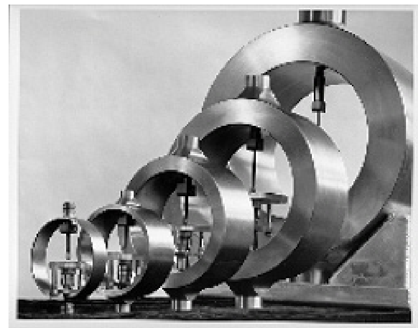


Fig. 1 Proving Rings of various sizes

Proving rings have evolved over time; however, they are still manufactured according to design specifications established in 1946 by the National Bureau of Standards (NBS), the predecessor of the National Institute of Standards and Technology (NIST). Those specifications can be found in the Circular of the National Bureau of Standards C454, issued in 1946. The concept behind the proving ring is illustrated in the diagram below.

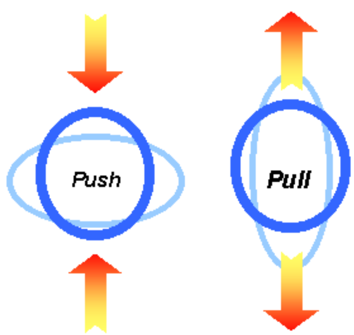
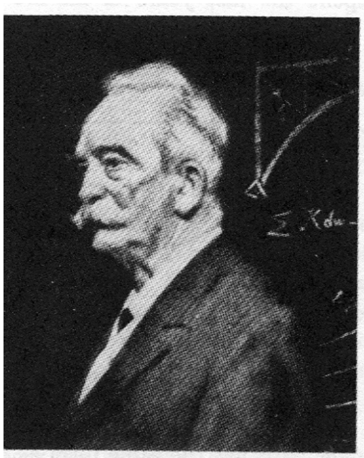


Fig. 2 Schematic diagram of the changes in the ring diameter as compression (push) and tension (pull) forces are applied.

Proving rings can be designed to measure either compression or tension forces. Some are designed to measure both. The basic operation of the proving ring in tension is the same as in compression. However, tension rings are provided with threaded bosses and supplied with pulling rods which are screwed onto the bosses.

Typically, proving rings are designed to have a deflection of about 0.84 mm to 4.24 mm. The relative measurement uncertainty can vary from 0.075 % to about 0.0125 %.



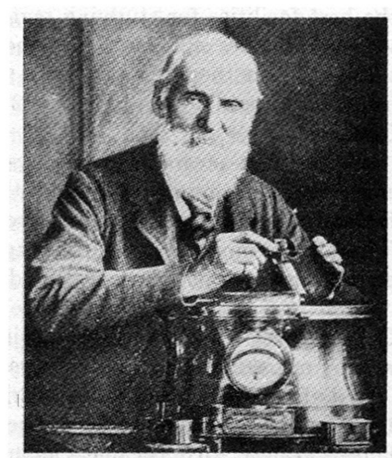
Otto Mohr (1835 – 1918)

Mohr designed some of the first steel trusses in Germany. Mohr was a very good professor, and his lectures aroused great interest in his students - the lectures were always clear and logically constructed.

Prominent students : Bach, Föppl.

Well known in Strength of Materials for his contribution of Mohr's circle. Until his time most engineers used maximum strain theory as propounded by Saint-Venant for designing structures. The tests of Bauschinger did not agree with maximum strain theory. Mohr initiated a systematic work on developing strength theories.

Lord Kelvin (1824 – 1907)



William Thomson at the age of 22 was elected professor of natural philosophy at Glasgow University. Thompson recognized the importance of experimental work for students. His laboratory was the first of its kind where students could learn through experimental work.

He introduced the notion of internal friction for elastic bodies. From his experiments he concluded that this friction is not proportional to velocity, as in fluids. He criticized the rariconstant theory of elastic constants, which was in vogue in France. He studied the temperature changes produced in elastic bodies which are subjected to strain, which lead to the development of strain gauges 80 years later!

Gave the first logical proof of the existence of a strain-energy function depending only upon the strain measured and not upon the manner in which strain is reached.

Measuring Strain On A Surface Through A Resistance Strain Gauge

A resistance strain gage is bonded to the surface of a component so well that it becomes an integral part of the component. Note that a strain gage is capable of measuring only the normal strain (tensile or compressive along the axis of the gage). To determine strain at a point a rosette is needed. The normal strain (ε) is related to change in electrical resistance ($\Delta R/R$) as

$S_g = (\Delta R/R)/ (\varepsilon)$

Where S_g is the gage factor provided by the manufacturer.

Strain is supposed to be measured at a point but experimentally a strain gage measures an average strain over an area. Bridge excitation, gage resistance, gage length, gage backing etc. influence the strain measurement. Wheatstone bridge is the most commonly used circuit for strain measurement.

Strain Measuring Bridge

Figure 1 shows a Wheatstone Bridge circuit. Initially E is adjusted to be zero by arranging the resistances such that $R_1 R_3 = R_2 R_4$. If, after this initial adjustment there are small changes in the values of the resistances, then the voltage output ΔE of the bridge can be obtained as:

$$\Delta E = V \frac{R_1 R_2}{\left(R_1 + R_2\right)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] \qquad \textbf{(1)}$$

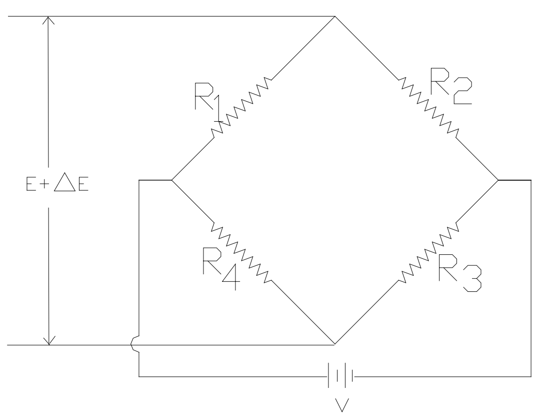


Fig. 1 Wheatstone Bridge

For optimum sensitivity it is recommended to have

R1 = R2 = R3 = R4

R1, R2, R3 and R4 can either be active strain gages or dummy strain gages.

The Eq. (1) can be interpreted as like strains in adjacent arms cancel but in opposite arms add. This aspect is judiciously used in strain measurement for temperature compensation or for doubling or quadrapling signal in transducer applications.

Applications of Wheatstone bridge to strain measurement

1. Single-arm bridge / Quarter Bridge Circuit

One of the arms of the bridge say R_1 is replaced by a strain gauge. With the initially balanced bridge, any change in resistance due to strain of the gauge causes a change in E. The output voltage obtainable from Eq.(1) is,

$$\Delta E = V\frac{R_1 R_2}{\left(R_1 + R_2\right)^2}\left[\frac{\Delta R_1}{R_1}\right] \tag{2}$$

In general, ΔR_1 can be due to both strain and temperature change. In order to measure the change in ΔR_1 only due to strain, the change in resistance due to temperature has to be cancelled. This is known as temperature compensation. Temperature compensation is critical for static measurements where the strain reading needs to be monitored for a long period of time.

2. Two–arm bridge / Half Bridge Circuit

A Wheatstone bridge that has two similar gauges in place of R_1 and R_2, while R_3 and R_4 built into the strain indicator, is termed as a Half bridge circuit. The output E is given by,

$$\Delta E = V\frac{R_1 R_2}{\left(R_1 + R_2\right)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right]$$

If both the gauges are in similar temperature environments, then the measurement is automatically compensated for temperature effects. In strain measurement it is *always recommended to use at least a half bridge circuit*. The strain sensitivity can be increased to two fold if choice of strain gage location are so selected such that ΔR_1 and ΔR_2 experience opposite but equal strains.

3. Four-arm Bridge / Full Bridge Circuit:

In this case all the four arms of the bridge are formed of similar strain gauge elements outside the strain indicator.
The strain sensitivity can be increased up to four times by judicious choice of pasting the strain gauges on the specimen and connecting them appropriately to the bridge.

Direct Reading – Strain indicator

A schematic diagram of the circuit element employed in a direct reading strain indicator is Fig. 2.

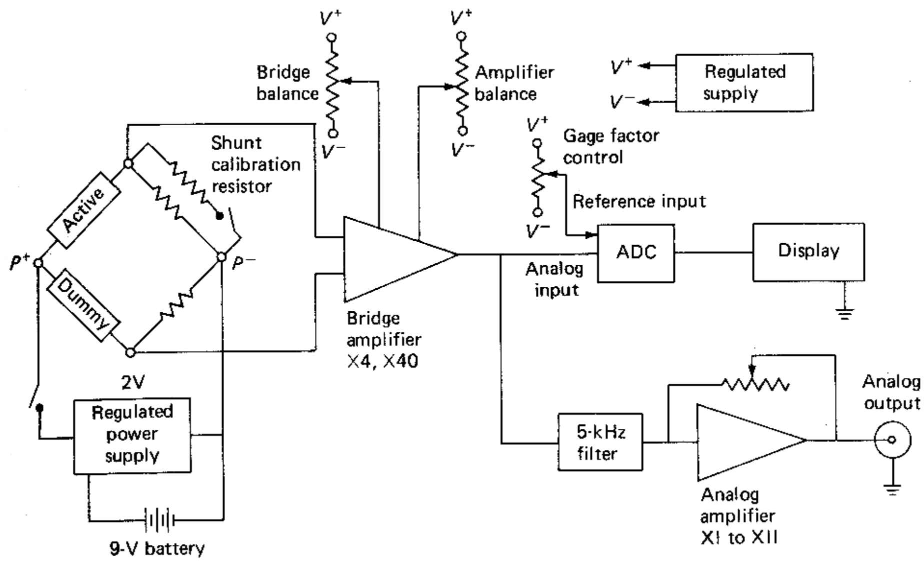


Fig. 2

- One can read directly the strain value and also take the output as an analog signal.
- Bridge is powered by a 2V dc supply.
- Shunt resistors produce a calibration indication of 5000 $\mu\varepsilon$ when the gage factor is set at 2.
- The output of the bridge is an input to an instrument amplifier
- A potentiometer is to balance the bridge. Note that this is used to adjust the voltage on the instrument amplifier rather than resistance on an arm of the bridge.
- Gage factor is adjusted through a potentiometer which controls the reference voltage of the analog-to-digital converter.

Model P-3500 Strain Indicator

Measurement of Stress - Introduction to Photoelasticity

Introduction

Photoelasticity is a whole field technique for visualizing and measuring stress fields in structures. It can directly give the field information of difference in principal stresses and its orientation θ.

The method utilizes a *bi-refringent* model (usually made of plastics) of the actual structure to view the stress contours due to external loading. The specimen under loading is viewed in an equipment called polariscope. When white light is used for illumination, a colorful fringe pattern reveals the stress distribution in the part. Qualitative analysis such as identification of stress concentration zones, uniform stress regions etc., can be done quite easily. Monochromatic light source for illumination enables better definition of fringes especially in areas with dense fringes and is usually recommended for manual data reduction.

Though Photoelastic experiment is conducted on plastics, the results are directly applicable for structures made of metals, as elastic constants do not influence stress and its distribution for problems of plane stress and strain when the body force is constant.

Photoelasticity can also be applied for the study of prototypes. This comes in handy to analyse problems such as measurement of assembly stresses and residual stresses. In such a case, a thin birefringent coating is pasted on the prototype and light reflected from the model is analysed.

Physical Principle Used in Photoelasticity

Certain non-crystalline transparent materials, notably some polymeric plastics are *optically isotropic* under normal circumstances but become *doubly refractive* (for a single incident ray two refracted rays are observed) or *bi-refringent* when stressed. This effect normally persists while the loads are maintained but vanishes almost instantaneously or after some interval of time depending on the material and conditions of loading when the loads are removed. Brewster first observed this phenomenon of temporary or artificial bi-refringence in 1816. This is the physical characteristic on which photoelasticity is based.

Formation of Fringes

Photoelasticity uses light as a sensor and fringe contours are formed along points where light transmitted through the model is extinct. Interpretation of the fringe pattern to meaningful representation of stress information is possible through the knowledge of crystal optics and associated mathematics. In this brief note we shall see only the salient results and for the derivation of the results one has to refer to detailed books on this subject.

The equipment used for Photoelastic analysis is called a polariscope. One uses basically a polarized beam of light for data interpretation and hence the name polariscope. Polarised light is obtained from a normal light source through the use of appropriate optical elements. The angular positions of the optical elements dictates the state of polarization of light. Since these elements can be rotated freely on their mounts, the polarization state of the incident light can be easily controlled.

Polarisation and Various States of Polarization

Most light sources consist of a large number of randomly oriented atomic or molecular emitters. The light rays emitted from such a source will have no preferred orientation and the tip of the light vector describes a random vibratory motion in a plane transverse to the direction of propagation. If the tip of the light vector is forced to follow a definite law, the light is said to be polarized.

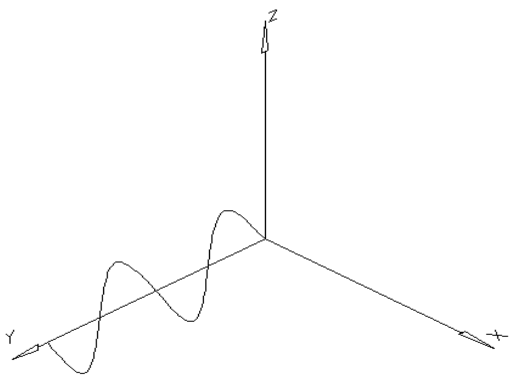


Fig.1 Linearly Polarised Light

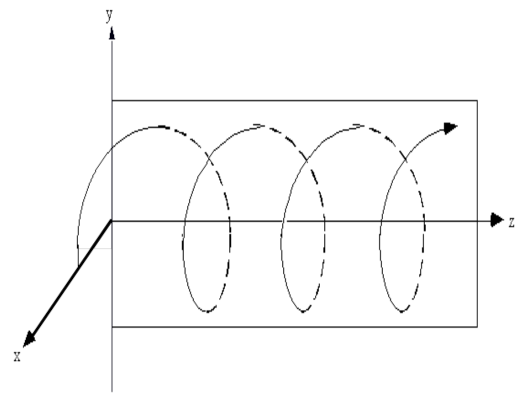


Fig.2 Circularly Polarised Light

For example, if the tip is constrained to lie on the circumference of a circle, it is said to be *circularly polarized*. If the tip describes an ellipse, it is said to be *elliptically polarised*. If the light vector is parallel to a given direction in the wave front, it is said to be *linearly* or *plane polarised*. Plane polarized light can be produced from a natural source by the following ways:

- Reflection
- Scattering
- Use of polarized sheet (Polariser, Analyser)
- Nicole's prism

Plane Polariscope

One of the simplest optical arrangements possible is the plane polariscope in which, a plane polarized light is incident on the model. A plane polariscope consists essentially of two polarizing elements called the polarizer and analyzer. Although they have different names, these elements are identical in nature and simply transmit plane-polarized light. The model to be analyzed is placed between them.

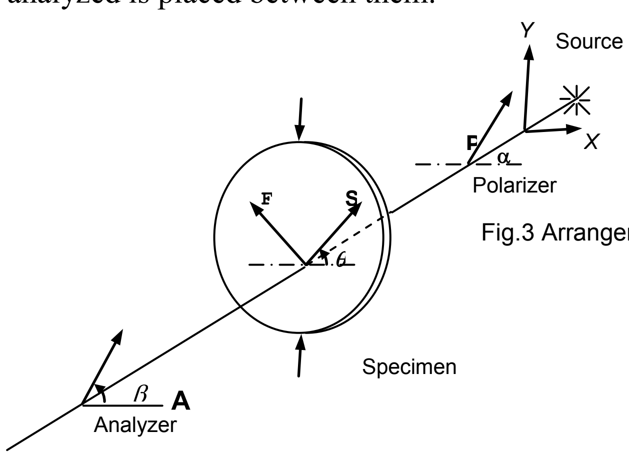


Fig.3 Arrangement for a Plane Polariscope

$\alpha = 90°$
$\beta = 0°$

In a plane polariscope, one observes two sets of contours namely isochromatics and isoclinics. When white light is used for illumination, isochromatics represent contours of constant color. An isochromatic fringe passes through points of equal principal stress difference and are usually numbered 0, 1, 2,… when the background field is dark.

When the polarizer axis coincides with one of the principal stress directions at the point of interest, one can observe a dark fringe even in white light. When the polarizer-analyser combination is rotated in unison, this black contour will also move. These are known as isoclinics, which means contours of constant inclination. Isoclinics are usually numbered with angles such as 0°, 10°,15° etc. The principal stress direction on all points lying on an isoclinic is a constant. The zeroth fringe order of isochromatics will also be black but this will not change when the polarizer-analyser combination is rotated.

Circular Polariscope

In this the incident light on the model is circularly polarized. Four optical elements are used in a circular polariscope. In the order in which the light pass through them are

a) A Plane-polarizing element called the polarizer
b) A quarter wave plate, which is followed by the model
c) A second quarter wave plate and finally
d) A second plane-polarizing element called the analyzer.

In a circular polariscope, one has only one set of contours namely Isochromatics. The background field could be either dark or bright. In dark field arrangement, fringes correspond to 0, 1, 2, … and in case of bright field arrangement, fringes correspond to 0.5, 1.5, 2.5,…etc.

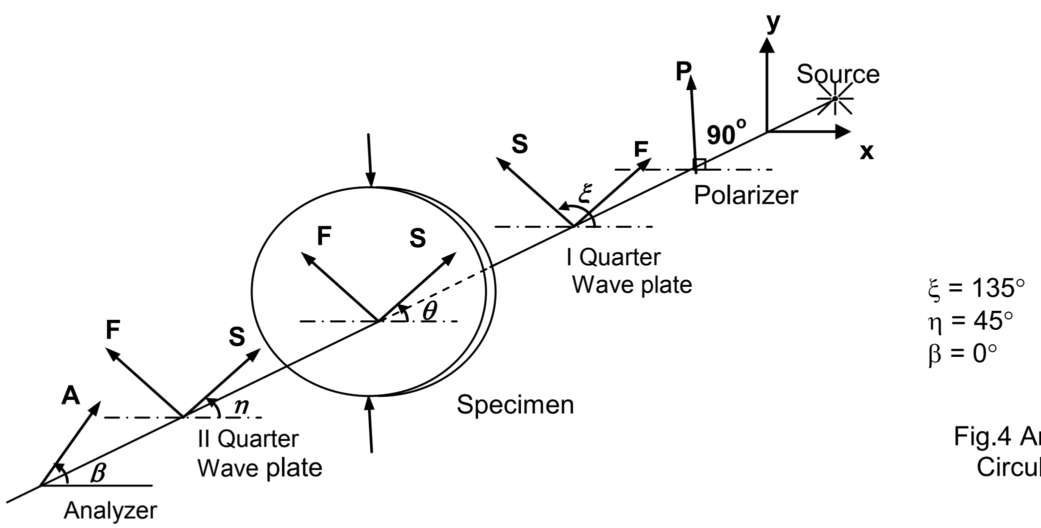


Fig.4 Arrangement for a Circular polariscope

Stress-Optic Law

Consider a transparent model made of high polymer subjected to a plane state of stress. Let the state of stress at a point be characterized by the principal stresses σ_1, σ_2 and their orientation with reference to a set of axes. When the model is stressed, experiments show that the model becomes doubly refractive and the directions of the polarizing axes in the plane of the model at any point coincide with the directions of the principal axes at that point.

Stress-Optic law gives the relation between the stress information and isochromatic fringe order *N*.

$$\sigma_1 - \sigma_2 = \frac{NF_\sigma}{t} \quad (1)$$

where, F_σ is the Material Stress Fringe value, N is the Fringe Order and t is the Thickness of the specimen.

Color Code

White light is useful if the isochromatic fringe orders are low i.e., 3 or less. Based on the colors observed one can identify the value of the isochromatic fringe order. Sensitivity is further augmented if observations are restricted to the positions of the purple bands — termed 'tints of passage' for the first three fringe orders. These occur at very narrow, sharply defined purple bands between red and blue fringes in the isochromatic pattern, and the positions of the tint-of-passage fringes can be ascertained with high precision.

Color Code is useful to identify fringe gradient direction. Look at the color code displayed in the Laboratory.

Report Writing

A report must be concise (but complete) and readable. Generally a report is typed but due to our limitations, it can be legibly handwritten preferably on bond papers. Attach the raw data to your report. Although this is not a general practice it helps to verify your calculation.

A laboratory report is generally divided into sub-headings like objective, theory, experimental details, results, discussion, conclusion and appendix. The objective is stated concisely in a sentence or two. For the experimental work of this laboratory, only the important definitions and theoretical results are to be presented. If you want to include a derivation, present it in an appendix and make a reference of it in the main text. The report is generally written for a technically trained person who may not be familiar with the work described in the report. Hence, some information on the background and the methodology adopted need to form part of the report. At the same time, in this busy world, one wants to reach the discussion and conclusion sections without spending too much time in knowing the background.

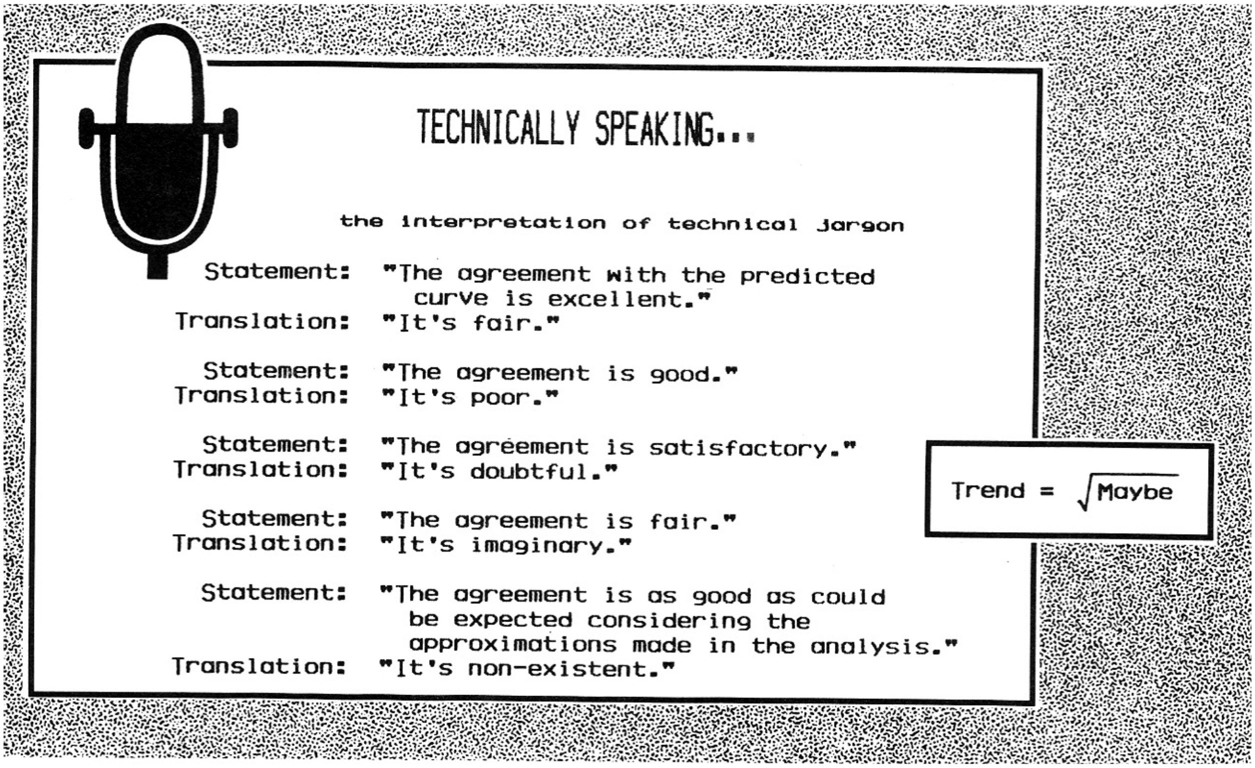
One of the best ways to discuss results is to compare them either with the results of other experimental results available in research papers or books or with theoretical predictions of a model. The comparison is generally found effective if presented in a graph or through a table.

It is very important to present the data in a readable table. Show sample calculations wherever necessary. Each table should be titled, too many columns crowd it and make it hard to read and therefore only the relevant portion of the raw data should be included. For example, initial reading of dial gauge is not carried to the formal text of the report; only the displacements are reported.

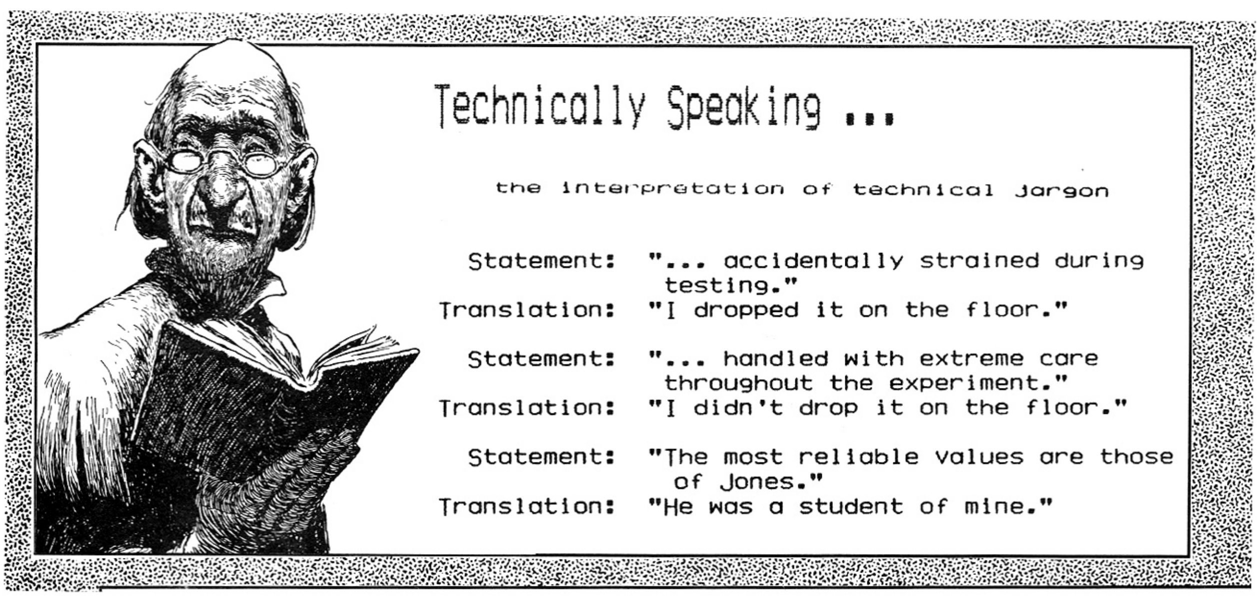
Graphs are very useful and are important features of a report to understand the results at a glance. Therefore, it pays to complete all the requirements of a graph; a suitable title should be stated clearly and if there are more than one line on the graph, each line should be identified properly. To make the graph readable, <u>simple scales</u> should be chosen for the axes and the axes are to be labeled with appropriate units.

Since tables and figures are the main parts of the report, they should be prepared with care and patience. All tables and figures (sketches, photographs and graphs) <u>must</u> be numbered, i.e., Table 1, Table 2 etc. and Fig.1, Fig. 2 etc. They do not become part of the report <u>until they are referred to in the text.</u> Only Figures and Tables referred in the text will be checked.

Since the writer spends hours in conducting the experiment and in reducing the data, he is in a good position to bring the highlights of the results to the attention of the reader. Also, the writer may discuss various aspects of the results such as limitations of the test equipments, problems arose during the experimentation, likely places of large errors etc.



Courtesy: Experimental Stress Analysis Note Book, Measurements Group, Inc., USA



Courtesy: Experimental Stress Analysis Note Book, Measurements Group, Inc., USA

How to Avoid Errors in Comparing Experimental Results with Theoretical Model

When you make measurements experimentally, the resulting value, if done carefully, is truth and can be used as a benchmark to test the analytical model. In many instances you may not measure force, torque or bending moment directly. You must clearly understand the functioning of the force application system and verify whether you have adjusted the necessary aspects suitably. If necessary draw the free body diagram of the system and check whether you are adopting a correct procedure in calculating the force/torque or bending moment as the case may be. If any distance measurement is involved, measure them with utmost care and always apply statistical principles in measurement. Take a few readings and use its average for any calculations.

Learn to be alert (difficult after a heavy lunch in the afternoon!) while doing the experiment. One should have a rough idea of what would be the trend of the experimental readings. For example if loading is increased, the deflection, stress or strain, should also increase in the same proportion. The results for loading and unloading should be close enough! If any deviation is observed you must make mid-course correction on the experimental procedure.

In the laboratory, the choice of dimensions of the test specimens are so chosen that it closely satisfies the assumptions made in the analytical model. Despite this, considerable deviations could exist if you do the experiment for the first time. One of the most common sources of error made by several students is the input that they feed to the analytical model. This input mainly consists of force, length, and cross-sectional details. These are to be measured and a careless approach in measurement could cost you dearly! For example if you do not measure the cross-sectional details of the specimen carefully, the error can blow up since $I = bd^3/12$ and any small error in d can blow up! On the other hand, the actual experimental measurement of deflection, strain or stress may be quite accurate yet the comparison is poor because input to analytical model is erroneous!

Appendix-1

Standard Test Method for Tension Testing of Metallic Materials

(E 8M-01)

(This section provides a taste of how the Codes – read. Continuity in the content is not focused but a flavour of how the codes are specific in their recommendation is highlighted)

1. **Scope:** To cover the tension testing of metallic materials in any form at room temperature, specifically, the methods of determination of yield strength, yield point elongation, tensile strength, elongation and reduction in area.

2 Reference Documents

3 Terminology:

3.1 **Discontinuous yielding**- a hesitation or fluctuation of force observed at the onset of plastic deformation, due to localized yielding. (The stress-strain curve need not appear to be discontinuous.)

3.2 **Lower yield strength, LYS**- the minimum stress recorded during discontinuous yielding, ignoring transient effects.

3.3 **Upper yield strength, UYS**- the first stress maximum (stress at first zero slope) associated with discontinuous yielding.

3.4 **Yield point elongation, YPE**- the strain (expressed in percent) separating the stress-strain curve's first point of zero slope from the point of transition from discontinuous yielding to uniform strain hardening. If the transition occurs over a range of strain, the YPE end point is the intersection between (a) a horizontal line drawn tangent to the curve at the last zero slope and (b) a line drawn tangent to the strain hardening portion of the stress-strain curve at the point of inflection. If there is no point at or near the onset of yielding at which the slope reaches zero, the material has 0 % YPE.

3.5 **Uniform elongation,**

4 Significance and Use: 5 Apparatus

6.Test specimens:

6.1 General:

6.1.1 *Specimen size*- Test specimens shall be either substantially full size or machined, as prescribed in the product specifications for the material being tested.

6.1.2 *Location*- Unless otherwise specified, the axis of the test specimen shall be located within the parent material as follows:

At the center for products 40mm or less in thickness, diameter or distance between the flats.

Midway from the center to the surface for products over 40mm in thickness, diameter or distance between flats.

6.1.3 *Specimen Machining*- Improperly prepared test specimens often are the reason for unsatisfactory and incorrect test results. It is important, therefore, that care be exercised in the preparation of specimens, particularly in the machining, to maximize precision and minimize bias in the test results.

The reduced sections of prepared specimens should be free of cold work, notches, chatter marks, grooves, gouges, burrs, rough surfaces or edges, overheating, or any other condition which may deleteriously affect the properties to be measured.

NOTE 1 – Punching or blanking of reduced section may produce significant cold work or shear burrs, or both, along the edges which should be removed by machining.

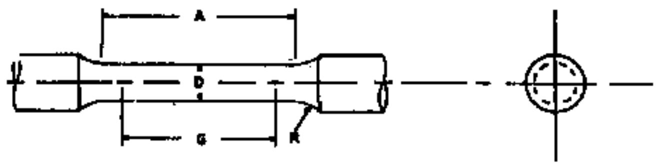
6.1.4 *Specimen Surface Finish*-

6.2 Round Specimens:

6.2.1 The standard 12.5-mm diameter round test specimen shown in Fig. 1 is used quite generally for testing metallic materials, both cast and wrought.

6.2.2 Fig. 1 also shows small-size specimens proportional to the standard specimen. These may be used when it is necessary to test material from which the standard specimen or specimens shown in Fig. 1 cannot be prepared. Other sizes of small, round specimens may be used. In any such small-size specimen, it is important that the gauge length for measurement of elongation be five times the diameter of the specimen.

6.2.3 The shape of the ends of the specimen outside of the gauge length shall be suitable to the material and of a shape to fit the holders or grips of the testing machine so that the forces may be applied axially. Fig. 2 shows specimens with various types of ends that have given satisfactory results.



	Dimensions, mm				
	Standard Specimen	Small-Size Specimens Proportional To Standard			
	12.5	9	6	4	2.5
G—Gage length	62.5 ± 0.1	45.0± 0.1	30.0 ± 0.1	20.0± 0.1	12.5 ± 0.1
D—Diameter (Note 1)	12.5 ± 0.2	9.0 ± 0.1	6.0 ± 0.1	4.0 ± 0.1	2.5 ± 0.1
R—Radius of fillet, min	10	8	6	4	2
A—Length of reduced section, min (Note 2)	75	54	36	24	20

NOTE 1—The reduced section may have a gradual taper from the ends toward the center, with the ends not more than 1 % larger in diameter than the center (controlling dimension).

NOTE 2—If desired, the length of the reduced section may be increased to accommodate an extensometer of any convenient gage length. Reference marks for the measurement of elongation should, nevertheless, be spaced at the indicated gage length.

NOTE 3—The gage length and fillets shall be as shown, but the ends may be of any form to fit the holders of the testing machine in such a way that the load may be axial (see Fig. 9). If the ends are to be held in wedge grips it is desirable, if possible, to make the length of the grip section great enough to allow the specimen to extend into the grips a distance equal to two thirds or more of the length of the grips.

NOTE 4—On the round specimens in Figs. 8 and 9, the gage lengths are equal to five times the nominal diameter. In some product specifications other specimens may be provided for, but the 5-to-1 ratio is maintained within dimensional tolerances, the elongation values may not be comparable with those obtained from the standard test specimen.

NOTE 5—The use of specimens smaller than 6 mm in diameter shall be restricted to cases when the material to be tested is of insufficient size to obtain larger specimens or when all parties agree to their use for acceptance testing. Smaller specimens require suitable equipment and greater skill in both machining and testing.

Fig. 1 Standard 12.5-mm Round Tension Test Specimen with Gauge Lengths Five Times the Diameters (5D), and Examples of Small-Sized Specimens Proportional to the Standard Specimen.

7 Procedures:

7.1 Preparation of the Test Machine

7.2 Measurement of Dimensions of Test Specimens

7.3 Gauge Length Marking of Test Specimens

7.4 Zeroing the Testing Machine:

7.5 Gripping of the Test Specimen:

7.6 Speed of Testing

7.7 **Determination of Yield Strength-**Determine yield strength by any of the methods described in the following paragraphs. Where extensometers are employed, use only those which are verified over a strain range in which the yield strength will be determined.

NOTE 13 - For example, a verified strain range of 0.2 to 2.0 % is appropriate for use in determining the yield strengths of many metals.

NOTE 14 - Determination of yield behavior on materials which cannot support an appropriate extensometer (thin wire. for example) is problematic and outside the scope of this standard.

7.7.1 *Offset Method*-To determine the yield strength by the offset method, it is necessary to secure data (autographic or numerical) from which a stress-strain diagram may be drawn. Then on the stress-strain diagram (Fig. 3) lay off *Om* equal to the specified value of the offset, draw parallel to *OA*, and thus locate r, the intersection of *mn* with the stress-strain diagram. In reporting values of yield strength obtained by this method, the specified value of offset used should be stated in parentheses after the term yield strength, as follows:

$$\text{yield strength (offset} = 0.2\ \%) = 360\ \text{MPa} \qquad (3)$$

In using this method, a Class B2 or better extensometer (see Practice E 83) shall be used.

NOTE 15 - There are two general types of extensometers, averaging and non-averaging, the use of which is dependent on the product tested. For most machined specimens, there are minimal differences. However, for some forgings and tube sections, significant differences in measured yield strength can occur. For these cases, it is recommended that the averaging type be used.

NOTE 16 - When there is a disagreement over yield properties, the offset method for determining yield strength is recommended as the referee method.

7.7.2 *Extension- Under-Load Method*-Yield strength by the extension-under-load method may be determined by: (1) using autographic or numerical devices to secure stress-strain data, and then analyzing this data(graphically or using automated methods) to determine the stress value at the specified value of extension, or (2) using devices that indicate when the specified extension occurs, so that the stress then occurring may be ascertained. Any of these devices may be automatic. This method is illustrated in Fig. 4. The stress at the specified extension shall be reported as follows:

$$\text{yield strength (EUL} = 0.5\ \%) = 360\ \text{MPa} \qquad (4)$$

Extensometers and other devices used in determination of the extension shall meet Class B2 requirements (see Practice 83) at the strain of interest, except where use of low-magnification Class C devices is helpful, such as in facilitating measurement of YPE if observed. If Class C devices are used, this must be reported along with the results.

NOTE 17 - The appropriate value of the total extension must be specified. For steels with nominal yield strengths of less than 550 MPa, an appropriate value is 0.005 mm/mm (0.5 %) of the gage length. For higher strength steels, a greater extension or the offset method should be used.

NOTE 18 - When no other means of measuring elongation are available, a pair of dividers or

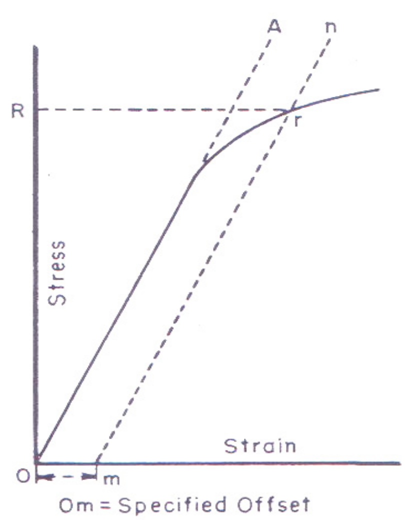


Fig. 3 Stress-Strain Diagram for Determination of Yield Strength by the Offset method

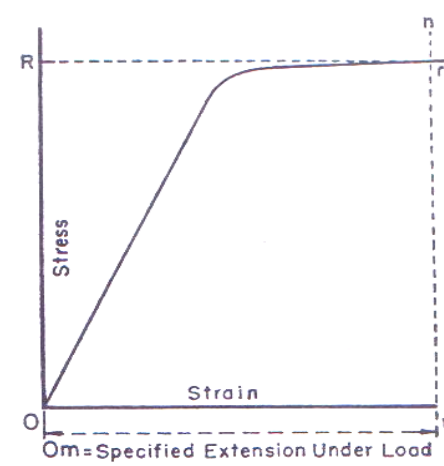


Fig. 4 Stress-Strain Diagram for Determination of Yield Strength by the Extension-Under-Load Method

similar device can be used to determine a point of detectable elongation between two gage marks on the specimen. The gage length shall be 50 mm. The stress corresponding to the load at the instant of detectable elongation may be recorded as the approximate extension-under-load yield strength.

7.7.3 *Autographic Diagram Method* (for materials exhibiting discontinuous yielding)- Obtain stress-strain (or force-elongation) data or construct a stress-strain (or load-elongation) diagram using an autographic device. Determine the upper or lower yield strength as follows:

7.7.3.1 Record the stress corresponding to the maximum force at the onset of discontinuous yielding as the upper yield strength. This is illustrated in Fig. 5 and Fig. 6.

NOTE 19 - If multiple peaks are observed at the onset of discontinuous yielding, the first is considered the upper yield strength. (See Fig. 5.)

7.7.3.2 Record the minimum stress observed during discontinuous yielding, (ignoring transient effects) as the lower yield strength. This is illustrated in Fig. 6.

NOTE 20 - Yield properties of materials exhibiting yield point elongation are often less repeatable and less reproducible than those of similar materials having no YPE. Offset and EUL yield strengths may be significantly affected by force fluctuations occurring in the region where the offset or extension intersects the stress-strain curve. Determination of upper or lower yield strengths (or both) may therefore be preferable for such materials, although these properties are dependent on variables such as test machine stiffness and alignment. Speed of testing may also have a significant effect, regardless of the method employed.

NOTE 21 - Where low-magnification autographic recordings are needed to facilitate measurement of yield point elongation for materials which may have discontinuous yielding. Class C extensometers may be employed. When this is done but the material exhibits no discontinuous yielding, the extension-under-load yield strength may be determined instead, using the autographic recording (see Extension-Under-Load Method).

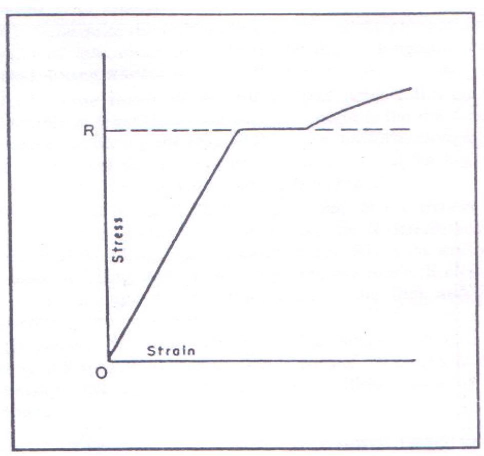


Fig. 5 Stress-Strain Diagram Showing Upper Yield Strength Corresponding with Top of Knee

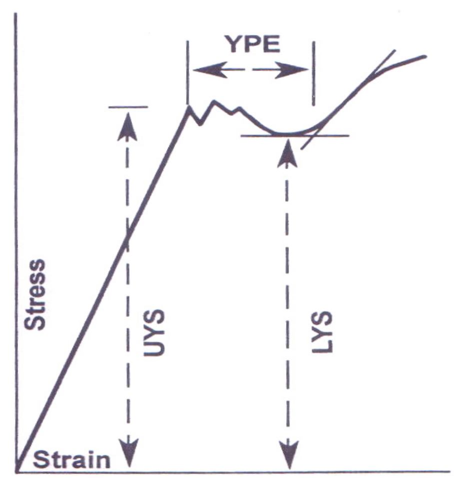


Fig. 6 Stress-Strain Diagram Showing Yield Point Elongation and Upper and Lower Yield Strengths.

7.7.4 *Halt-of-the-Force Method* (for materials exhibiting discontinuous yielding)- Apply an increasing force to the specimen at a uniform deformation rate. When the force hesitates, record the corresponding stress as the upper yield strength.

NOTE 22 - The Halt-of-the-Force Method was formerly known as the Halt-of-the-Pointer Method, the Drop-of-the-Beam Method, and the Halt-of-the-Load Method.

7.8 Yield Point Elongation

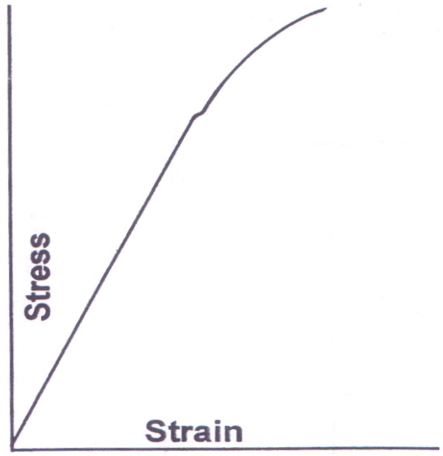


Fig. 7 Stress-Strain Diagram With an Inflection, But No YPE

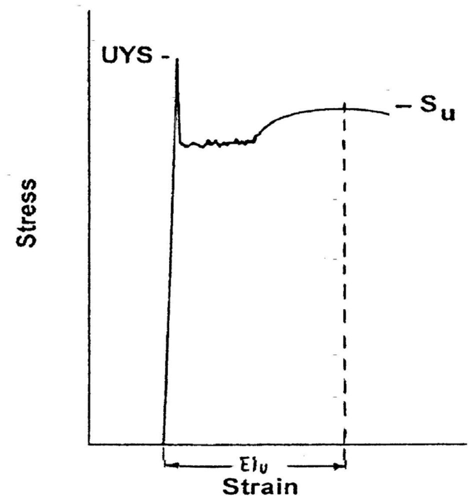


Fig. 8 Stress-Strain Diagram in Which the Upper Yield Strength is the Maximum Stress Recorded

7.9 Uniform Elongation

7.10 Tensile Strength-Calculate the tensile strength by dividing the maximum force carried by the specimen during the tension test by the original cross-sectional area of the specimen.

NOTE 37 – If the upper yield strength is the maximum stress recorded, and if the stress-strain curve resembles that of Fig. 8, it is recommended that the maximum stress *after discontinuous yielding* be reported as the tensile strength. Where this may occur, determination of tensile strength should be in accordance with the agreement between the parties involved.

X1 Factors Affecting Tension Test Results

X1.1 The precision and bias of tension test strength and ductility measurements depend on strict adherence to the stated test procedure and are influenced by instrumental and material factors, specimen preparation, and measurement/testing errors.

X1.2 The consistency of agreement for repeated tests of the same material is dependent on the homogeneity of the material, and the repeatability of specimen preparation, test conditions, and measurements of the tension test parameters.

X1.3 Instrumental factors that can affect test results include: the stiffness, damping capacity, natural frequency, and mass of moving parts of the tensile test machine; accuracy of force indication and use of forces within the verified range of the machine; rate of force application, alignment of the test specimen with the applied force, parallelness of the grips, grip pressure, nature of the force control used, appropriateness and calibration of extensometers, heat dissipation (by grips, extensometers, or ancillary devices), and so forth.

X1.4 Material factors that can affect test results include: representativeness and homogeneity of the test material, sampling scheme, and specimen preparation (surface finish, dimensional accuracy, fillets at the ends of the gage length, taper in the gage length, bent specimens, thread quality, and so forth).

X1.4.1 Some materials are very sensitive to the quality of the surface finish of the test specimen (see Note 8) and must be ground to a fine finish, or polished to obtain correct results.

X1.4.2 Test results for specimens with as-cast, as-rolled, as-forged, or other non-machined surface conditions can be affected by the nature of the surface (see Note 15).

X1.4.3 Test specimens taken from appendages to the part or component, such as prolongs or risers, or from separately produced castings (for example, keel blocks) may produce test results that are not representative of the part or component.

X1.4.4 Test specimen dimensions can influence test results. For cylindrical or rectangular specimens, changing the test specimen size generally has a negligible effect on the yield and tensile strength but may influence the upper yield strength, if one is present, and elongation and reduction of area values. Comparison of elongation values determined using different specimens requires that the following ratio be controlled:

$$L_o / (A_0)^{1/2} \qquad \text{(XI.1)}$$

where: L_O is original gage length of specimen, and A_O is original cross-sectional area of specimen.

X1.4.4.1 Specimens with smaller $Lo/(A_0)^{1/2}$, ratios generally give greater elongation and reduction in area values. This is the case, for example, when the width or thickness of a rectangular tensile test specimen is increased.

X1.4.4.2 Holding the $Lo/(A_O)^{1/2}$ ratio constant minimizes, but does not necessarily eliminate, differences. Depending on material and test conditions, increasing the size of the proportional specimen of Fig. 8 may be found to increase or decrease elongation and reduction in area values somewhat.

X1.4.5 Use of a taper in the gage length, up to the allowed 1 % limit, can result in lower elongation values. Reductions of as much as 15 % have been reported for a 1 % taper.

X1.4.6 Chances in the strain rate can affect the yield strength, tensile strength, and elongation values, especially for materials, which are highly strain rate sensitive. In general, the yield strength and tensile strength will increase with increasing strain rate, although the effect on tensile strength is generally less pronounced. Elongation values generally decrease as the strain rate increases.

X1.4.7 Brittle materials require careful specimen preparation, high quality surface finishes, large fillets at the ends of the gage length, oversize threaded grip sections, and cannot tolerate punch or scribe marks as gage length indicators.

X1.4.8 Flattening of tubular products to permit testing does alter the material properties, generally nonuniformity, in the flattened region, which may affect test results.

X1.5 Measurement errors that can affect test results include: verification of the test force, extensometers, micrometers, dividers, and other measurement devices, alignment and zeroing of chart recording, devices, and so forth.

X1.5.1 Measurement of the dimensions of as-cast, as-rolled, as-forged, and other test specimens with non-machined surfaces may be imprecise due to the irregularity of the surface flatness.

X1.5.2 Materials with anisotropic flow characteristics may exhibit noncircular cross sections after fracture and measurement precision may be affected, as a result (see Note 37).

X1.5.3 The corners of rectangular test specimens are subject to constraint during deformation and the originally flat surfaces may be parabolic in shape after testing which will affect the precision of final cross-sectional area measurements (see Note 42).

X1.5.4 If any portion of the fracture occurs outside of the middle of the gage length or in a punch or scribe mark within the gage length, the elongation and reduction of area values may not be representative of the material. Wire specimens that break at or within the grips may not produce test results representative of the material.

X1.5.5 Use of specimens with shouldered ends ("button- head" tensiles) will produce lower 0.02 % offset yield strength values than threaded specimens.

X1.6 Because standard reference materials with certified tensile property values are not Available, it is not possible to rigorously define the bias of tension tests. However, by the use of carefully designed and controlled interlaboratory studies, a reasonable definition of the precision of tension test results can be obtained.

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References

1. Crandall, Dahl, and Lardner, "An Introduction to The mechanics of Solids", McGraw-Hill, 1978.
2. S. P. Timoshenko, "History of Strength Of materials", Dover Publications, 1953.