MA2030: Linear Algebra and Numerical Analysis
Assignment-6

January – May 2012

1. Consider $\mathbb{R}^3$ with the standard inner product. In each of the following, find orthogonal vectors obtained from the given vectors using Gram-Schmidt orthogonalization procedure:

   (a) $(1, 2, 0), \quad (2, 1, 0), \quad (1, 1, 1)$

   (b) $(1, 1, 1), \quad (1, -1, 1), \quad (1, 1, -1)$

   (c) $(0, 1, 1), \quad (0, 1, -1), \quad (-1, 1, -1)$.

2. Consider $\mathbb{R}^3$ with the standard inner product. In each of the following, find a vector of norm 1 which is orthogonal to the given two vectors:

   (a) $(2, 1, 0), \quad (1, 2, 1)$

   (b) $(1, 2, 3), \quad (2, 1, -2)$

   (c) $(0, 2, -1), \quad (-1, 2, -1)$.

   [Hint: If $\{u_1, u_2\}$ is linearly independent, then find a vector u_3 so that $\{u_1, u_2, u_3\}$ is also linearly independent; use Gram-Schmidt orthogonalization procedure. Alternatively, Find α, β, γ such that $u_3 = (\alpha, \beta, \gamma)$ satisfies $\langle u_1, u_3 \rangle = 0$ and $\langle u_2, u_3 \rangle = 0$. Next, normalize u_3.]

3. Consider $\mathbb{R}^4$ with the standard inner product. In each of the following, find $V_0^\perp$ where $V_0 = \text{span}\{u_1, u_2\}$:

   (a) $u_1 = (1, 2, 0, 1), \quad u_2 = (2, 1, 0, -1)$

   (b) $u_1 = (1, 1, 1, 0), \quad u_2 = (1, -1, 1, 1)$

   (c) $u_1 = (0, 1, 1, -1), \quad u_2 = (0, 1, -1, 1)$.

   [Hint: Extend $\{u_1, u_2\}$ to a basis $\{u_1, u_2, u_3, u_4\}$; obtain orthogonal vectors v_1, v_2, v_3, v_4 by Gram-Schmidt orthogonalization procedure. Then $V_0^\perp = \text{span}\{v_3, v_4\}$.]

4. Consider $\mathbb{R}^{3\times 3}$ with the inner product $\langle A, B \rangle = \text{trace}(B^T A)$. Using Gram-Schmidt orthogonalization procedure, find a nonzero matrix which is orthogonal to both the matrices

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

5. Consider the polynomials $u_j(t) = t^{j-1}$ for $j = 1, 2, 3$ in the vector space of all polynomials with real coefficients. By Gram-Schmidt orthogonalization procedure, find orthogonal polynomials obtained from u_1, u_2, u_3 with respect to the following inner products:

   (a) $\langle p, q \rangle = \int_0^1 p(t)q(t)\,dt$

   (b) $\langle p, q \rangle = \int_{-1}^1 p(t)q(t)\,dt$

   (c) $\langle p, q \rangle = \int_{-1}^0 p(t)q(t)\,dt$.