MA 2020 . Quiz 1 . Mark Scheme

Ans 1. The given differential equation can be written as $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2 log\ x}{x}$.
Dividing by y^2 on both sides and substituting $z=\frac{1}{y}$, we get
$\frac{dz}{dx}-\frac{z}{x}=-\frac{log\ x}{x^2}$. [1]

Therefore, using the method for non-linear

$$\begin{aligned} \frac{z}{x} &= -\int \frac{log\ x}{x^2}dx + c \\ &= \frac{1+log\ x}{x}+c \\ &= \frac{1+log\ x+cx}{x}. \end{aligned}$$

Therefore

$$y=\frac{1}{1+log\ x+cx}.$$

One mark for arriving at the first line in the above array and then one mark for completing the rest.

Remark 1. If one has made a sign error in the above mentioned integration and the final answer is correct modulo the sign, one mark is cut.

Ans. 2 $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=\frac{3}{x}$ [1]

Therefore, the integrating factor $\mu=x^3$. [1]
Therefore $u=\int M dx+\phi(y)=\frac{1}{2}x^4\sin y^2+\phi(y)$ [1]
Comparing with $\frac{\partial u}{\partial y}=N=x^4 y\cos y^2$, one gets $\phi'(y)=0$ and hence the solution is $x^4\sin y^2=c$. [1]

Ans. 3 $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}-\frac{2y}{x^2}=0$. Given that $y_1(x)=x$. Then

$y_2=y_1\int\frac{1}{y_1^2}e^{-\int a_1 dx}dx$ [1]

$=x\int\frac{1}{x^4}dx$ [1]

$=\frac{-1}{3x^2}$.

Therefore, the general solution is $y=c_1x+\frac{c_2}{x^2}$ [1]

Q. No. 4:

(General solution by the method of variation of parameters):

Solutions $sin\ x$ and $cos\ x$ for the homogeneous equation. (1 mark)

Showing that the Wronskian equals 1. (1 mark)

$A'(x) = -1$ and $B'(x) = cot\ x$. (1 mark)

(Here a particular integral is sought in the form $y = A(x) cos\ x + B(x) sin\ x$)

$A(x) = -x + c_1,\ B(x) = log\ sin\ x + c_2$. (1 mark)

General solution. (1 mark)

Remarks 0.1. *When a student obtains correct expressions for one of $A'(x)$ and $B'(x)$ as above, but not both, and goes on to obtain the correct expression after integration, he gets three marks. A student who obtains A and B correctly, but writes four constants in the general solution, forfeits one mark.*

Q. No. 5:

(Solution of simultaneous equations):

Obtaining the equation $y'' - 4y' - 5y' = 0$ for y_1 or y_2. (1 mark)

$y_1(x) = Ae^{-x} + Be^{5x}$. (1 mark)

$y_2(x) = -Ae^{-x} + \frac{1}{2}Be^{5x}$. (1 mark)

$A = -1,\ B = 2$. (2 marks)

Remarks 0.2. *Students making a mistake in the auxiliary equation (the characteristic equation) get no marks. Students obtaining correct eigen values and correct eigen vectors, but do not write the correct expressions for y_1 or y_2 get three marks.*