Assignment Sheet 1
MA2020 Differential Equations (July - November 2012)

1. Solve the following first order differential equations

   (a) $x\frac{dy}{dx} + y = x^3y^6$

   (b) $xy^2\frac{dy}{dx} + y^3 = x\cos(x)$

   (c) $x\frac{dy}{dx} + y = y^2 log(x)$

   (d) $(x^2y^3 + xy)\frac{dy}{dx} = 1$

   (e) $\frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1 + y^2) = 0$

   (f) $(x^2 + y)dx + (y^3 + x)dy = 0$

   (g) $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$

   (h) $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$

   (i) $(xdx + ydy)(x^2 + y^2) = ydx - xdy$

   (j) $y\cos(x)dx + 2\sin(x)dy = 0$

   (k) $xdy + ydx + 3x^3y^4dy = 0$

   (l) $(1 + xy)ydx + (1 - xy)xdy = 0$

   (m) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

   (n) $(xy - 1)dx + (x^2 - xy)dy = 0$

2. Find the curve passing through the point $(1, 0)$ and having at each of its points the slope $-\frac{x}{y}$

3. Solve the following Initial Value Problems (IVP)

   (a) $\frac{dy}{dx} = 2y + e^{2x}$, $y(0) = 3$

   (b) $\frac{dy}{dx} = 3y + 2e^{3x}$, $y(0) = 2$

   (c) $\frac{dy}{dx} = y\tan(x) + \sec x$, $y(0) = -1$

   (d) $\frac{dy}{dx} = \frac{2}{x}y + x$, $y(1) = 2$

4. Define the Wronskian $w(y_1, y_2)$ of any two continuous function y_1 and y_2 defined in an interval $(a, b) \subset R$. Show that $w(y_1, y_2) = 0$ if y_1 and y_2 are linearly dependent.

5. If y_1 and y_2 are any two solutions of a second order linear homogeneous ordinary differential equation which has been defined in an interval $(a, b) \subset R$, then $w(y_1, y_2)$ is either identically zero or non-zero at any point of the interval (a, b).

6. If y_1 and y_2 are two linearly independent solutions of a second order linear homogeneous ordinary differential equation then prove that $y = c_1y_1 + c_2y_2$, where c_1 and c_2 are constants, is a general solution.

7. Find the general solution of the following second order equations using the given known solution y_1.

   (a) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y \;=\; 0$ where $y_1(x) = x$.

   (b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y \;=\; 0$ where $y_1(x) = x^2$.

   (c) $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y \;=\; 0$ where $y_1(x) = x$.

   (d) $x\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + (x+1)y \;=\; 0$ where $y_1(x) = e^x$.

8. Find the general solution of each of the following equations ($D^n \equiv \frac{d^n}{dx^n}$)

   (a) $(D^4 - 81)y = 0$

   (b) $(D^3 - 4D^2 + 5D - 2)y = 0$

   (c) $(D^4 - 7D^3 + 18D^2 - 20D + 8)y = 0$

   (d) $(D^4 + 2D^3 + 3D^2 + 2D + 1)y = 0$

   (e) $(D^2 - 3D - 6)y \;=\; 3\sin 2x$

   (f) $(D^2 + 1)y \;=\; 2\cos x$

   (g) $(D^2 - 3D + 2)y \;=\; (4x+5)e^{3x}$

   (h) $(D^2 - 1)y \;=\; 3e^{2x}\cos 2x$

   (i) $(D^2 - 2D + 3)y \;=\; 3e^{-x}\cos x$

9. Solve the following using the method of variation of parameters ($D^n \equiv \frac{d^n}{dx^n}$)

   (a) $(D^2 + 1)y \;=\; cosecx$

   (b) $(D^2 - D - 6)y \;=\; e^{-x}$

   (c) $(D^2 + a^2)y \;=\; \tan ax$

   (d) $(D^2 + a^2)y \;=\; \sec 2x$

   (e) $x^2y'' - 4xy' + 6y \;=\; 21x^{-4}$

   (f) $4x^2y'' + 8xy' - 3y \;=\; 7x^2 - 15x^3$

   (g) $x^2y'' - 2xy' + 2y \;=\; x^3\cos x$

   (h) $xy'' - y' \;=\; (3+x)x^2e^x$

10. Solve the following problems using method of undetermined coefficients ($D^n \equiv \frac{d^n}{dx^n}$)

   (a) $(D^3 - 2D^2 - 5D + 6)y \;=\; 18e^x$

   (b) $(D^2 + 25)y \;=\; 50\cos 5x + 30\sin 5x$

   (c) $(D^3 - 2D^2 + 4D - 8)y \;=\; 8(x^2 + \cos 2x)$

   (d) $(D^3 + 3D^2 - 4)y \;=\; 12e^{-2x} + 9e^x$

11. Show that the general solution of $L(y) = f_1(x) + f_2(x)$, where $L(y)$ is a linear differential operator, is $y = \bar{y} + y^*$, where $\bar{y}$ is the general solution of the $L(y) = 0$ and y^* is the sum of the any particular solutions of $L(y) = f_1(x)$ and $L(y) = f_2(x)$.