Assignment Sheet 2
MA2020 Differential Equations (July - November 2012)

1. Find the general (or unique) solution of the following system of equations

   (a) $\frac{dy_1}{dx} = -3y_1 + 4y_2 \qquad \frac{dy_2}{dx} = -2y_1 + 3y_2$

   (b) $\frac{dy_1}{dx} = 2y_1 - y_2 \qquad \frac{dy_2}{dx} = 3y_1 + 6y_2$ and $y_1(0) = 2,\ y_2(0) = -10$

   (c) $\frac{dy_1}{dx} = y_1 - 2y_2 \qquad \frac{dy_2}{dx} = 4y_1 + 5y_2$

   (d) $\frac{dy_1}{dx} = 5y_1 + 4y_2 \qquad \frac{dy_2}{dx} = -y_1 + y_2$

   (e) $\frac{dy_1}{dx} = 3y_1 + 4y_2 \qquad \frac{dy_2}{dx} = 2y_1 + y_2$ and $y_1(0) = 1,\ y_2(0) = 2$

   (f) $\frac{dy_1}{dx} = 7y_1 + 6y_2 \qquad \frac{dy_2}{dx} = 2y_1 + 6y_2$

   (g) $\frac{dy_1}{dx} = -4y_1 - y_2 \qquad \frac{dy_2}{dx} = y_1 - 2y_2$

   (h) $\frac{dy_1}{dx} = 5y_1 + 2y_2 + 5x \qquad \frac{dy_2}{dx} = 3y_1 + 4y_2 + 17x$

2. Find the power series solutions of the following second order linear equations

   (a) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2+1)y = 0$ about $x_0 = 0$

   (b) $(x^2-1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0\ y(0) = 4,\ y'(0) = 6$

   (c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ about $x_0 = 1$

   (d) $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 0$ about $x_0 = 1$

   (e) $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0,\ \ y(1) = 2,\ \ y'(1) = 4$

   (f) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0,\ \ y(2) = 0,\ y'(2) = \frac{1}{e}$

3. Find two linearly independent solutions of

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

   about $x_0 = 0$, where n is a constant.

4. Find a polynomial approximation of the fourth degree to the solution of the following initial value problems

   (a) $(1+2x)\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0,\ \ y(0) = 0,\ \ y'(0) = 1$

   (b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (1+x)y = 0,\ \ y(0) = -1,\ \ y'(0) = 0$

   (c) $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0,\ \ y(1) = 2,\ \ y'(1) = 4$

   (d) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0,\ \ y(2) = 0,\ y'(2) = \frac{1}{e}$