(c) $T : \mathbb{M}_{2\times 3} \to \mathbb{M}_{2\times 2}$;

$$T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}$$

(d) $T : \mathrm{P}_2(\mathbb{R}) \to \mathrm{P}_3(\mathbb{R})$; $\quad T(f(x)) = xf(x) + f'(x)$.

(e) $T : \mathbb{M}_{n\times n}(\mathbb{R}) \to \mathbf{R}$;
$T(A) = tr(A)$, where $tr(A) = \sum_{i=1}^{n} a_{ii}$ and $A = (a_{ij})_{n\times n}$.

9. Let V, W be vector space over $\mathbb{R}$ and $T : V \to W$ be linear. Prove that if V is finite dimensional, then $N(T)$ and $R(T)$ are finite dimensional and dimension of V =rank of T+ nullity of T.

10. Let V, W, T be as in the last problem with $Dim(V) = Dim(W)$. Prove that T is $1-1$ if and only if T is onto.

11. Prove that row rank of a matrix A equals its column rank.

12. Let V and W be finite dimensional vector space and $T : V \to W$ be linear.

    (a) Prove that if $\dim(V) < \dim(W)$, then T cannot be onto.

    (b) Prove that if $\dim(V) > \dim(W)$, then T cannot be one-to-one.

13. (a) Give an example of distinct linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $N(T) = R(T)$.

    (b) Give an example of distinct linear transformations T and U such that $N(T) = N(U)$ and $R(T) = R(U)$.

    (c) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined as $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let B be the standard ordered basis for $\mathbf{R}^2$, $C = \{(1,1,0), (0,1,1), (2,2,3)\}$ and $D = \{(1,2), (2,3)\}$.Compute $[T]_B^C$, $[T]_D^C$.

14. For the following parts,
let $\alpha = \{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\}$, $\beta = \{1, x, x^2\}$, $\gamma = \{1\}$

    (a) Define $T : \mathbb{M}_{2\times 2}(\mathbf{R}) \to \mathbf{R}_{2\times 2}(\mathbf{R})$ by $T(A) = A^t$. Compute $[T]_\alpha$.

    (b) Define $T : \mathbf{P}_2(\mathbf{R}) \to \mathbb{M}_{2\times 2}(\mathbf{R})$ by $T(f) = \begin{pmatrix} f'(0) & 2f(1) \\ 0 & f'(3) \end{pmatrix}$
    , compute $[T]_\beta^\alpha$.

    (c) Define $T : \mathbb{M}_{2\times 2}(\mathbf{R}) \to \mathbf{R}$ by $T(A) = tr(A)$. Compute $[T]_\alpha^\gamma$.

    (d) Define $T : \mathbf{P}_2(\mathbf{R}) \to \mathbf{R}$ by $T(f) = f(2)$. Compute $[T]_\beta^\gamma$.