Assignment-10

1. Show that the eigenvalues of a real skew symmetric matrix are purely imaginary.
2. Show that the eigenvalues of a real orthogonal matrix have absolute value 1.
3. Show that the eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are orthogonal.
4. If x and y are eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix of order 3, then show that the cross product of x and y is a third eigenvector linearly independent with x and y.
5. Diagonalize the following matrices:

$$(a) \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad (b) \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

Assignment-10

6. Give examples of matrices which cannot be diagonalized.
7. Which of the following linear transformation T is diagonalizable? If it is diagonalizable, find the basis E and the diagonal matrix $[T]_{E,E}$.
   (a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3,\ x_1 + x_2 - x_3,\ x_1 - x_2 + x_3)$.

   (b) $T : \mathcal{P}_3 \to \mathcal{P}_3$ such that $T(a_0 + a_1 t + a_2 t^2 + a_3 t^3) = a_1 + 2a_2 t + 3a_3 t^2$.

   (c) $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $Te_1 = 0,\ Te_2 = e_1,\ Te_3 = e_2$.
   (d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $Te_1 = e_2,\ Te_2 = e_3,\ Te_3 = 0$.
   (e) $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $Te_1 = e_3,\ Te_2 = e_2,\ Te_3 = e_1$.

Assignment-10

8. Check whether the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ corresponding to each of the following matrix is diagonalizable. If diagonalizable, find a basis of eigenvectors for the space $\mathbb{R}^3$:

$$(a) \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$(d) \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 3/2 & 0 \\ 1/2 & -1/2 & 1 \end{bmatrix} \qquad (e) \begin{bmatrix} 3 & -1/2 & -3/2 \\ 1 & 3/2 & 3/2 \\ -1 & -1/2 & 5/2 \end{bmatrix}$$

Assignment-11

1. Consider the system of linear equations:
   $x_1 - x_2 + 2x_3 - 3x_4 = 7$, $4x_1 + 3x_3 + x_4 = 9$,
   $2x_1 - 5x_2 + x_3 = -2$, $\quad 3x_1 - x_2 - x_3 + 2x_4 = -2$.
   Connect the ranks with the process of Gaussian Elimination. Determine the solution set of the system.
2. Using Gaussian Elimination, find all $k \in \mathbb{R}$ such that the system of linear equations:
   $x + y + 2z - 5w = 3$, $\quad 2x + 5y - z - 9w = -3$,
   $x - 2y + 6z - 7w = 7$, $\ 2x + 2y + 2z + kw = -4$
   has more than one solution.